

EUROPEAN CALL OPTION: PRICING UNDER PRESSURE

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Abstract

Upon today's economic activity and the uncertain feature of stocks, predicting the return profit of an investment on the stock market will be a challenging task. Regardless of the challenge, many financial institutes today provide a consulting service for investors who are planning to focus their business on the stock market. These institutes use advanced mathematical prediction tools to study and forecast the prices associated with the stock market. Many would agree that running such a forecast plays a great role in building a well-designed and more stable financial firm.

This paper presents a case study of Client Services Division of FirstBank, a large investment bank providing brokerage services to clients across the United States. And the project was undertaken to do analysis for a client who is interested in purchasing a European call option based on the data provided. This call option will entitle the client the right to purchase shares of stock.

In this thesis project, simulated mathematical models of pricing an option will be used in predicting the value of European call option. The efficiency and accuracy of the models will be checked by taking a number of call options traded in the market recently. The study will price the option in today's dollars with a better model and will provide a recommendation in exercising the call option.

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1. Introduction

This project centers on the case study published by Hillier and Lieberman titled, *Pricing Under Pressure* [16]. Elise Sullivan, Analyst working in the client services Division of FirstBank, was given her first assignment to price an option for a client named Emery Bowlander. FirstBank is a large investment bank providing brokerage services to clients across the United States. Emer Bowlander is an investor interested in purchasing a European call option that provides him with the right to purchase shares of Fellare stock for \$44.00 on the first of February-12 weeks from today. Fellare is an aerospace manufacturing company operating in France, and Mr. Bowlander has a strong feeling that the European Space Agency will award Fellare with a contract to build a portion of the International Space Station sometime in January.

Mr. Bowlander believes that his investment is worth if the European Space Agency awards the contract to Fellare. By taking into account of the financial risk associated with the stock, he expects a figure on the price of the call option before the stock market closes. The figure will help him in deciding whether to exercise the option or not.

This project will attempt to price the European call option for Fellare Stock based on the information provided on the case study. Different approaches in pricing the call option will be presented and a simulation model will be developed. The precision level of the approaches and the models will be examined using a number of real market option prices. In addition the movement of the stock market will be studied. Pricing the option being the main goal of this project, recommendations will also be provided after running the financial risk analysis associated with the call option. And the predicted price of the call option will play a great role for the investor in making a valuable decision in his future investment.

1. Options on Stock

Making investment in the stock market requires a basic understanding of how derivatives work. As it might be a win-lose situation, one need to be able to identify the risk associated with the type of derivative in practice ahead of time. Derivative is a financial instrument (or more simply, an agreement between two people) that has a value determined by the price of something else. [14] An option by itself is a type of derivative that needs to be priced before its exercising time. An option on a stock is a security that gives the holder the right to buy or to sell one share of the stock on or before a particular date for a predetermined price. [15] In addition to the current economic situation, being able to forecast the price of an option on a stock market will enable an investor to have a first look into the financial risk associated with that specific stock.

Today many financial institutes implement a sophisticated method of pricing an option before exercising the option on the stock market. In this paper I will present the different methods of option pricing, ranging from the arbitrage method to the Monte-Carol method. The Noble prize winning formula, Black–Scholes formula, will also be presented.

Before I start my presentation on the different methods of pricing an option, below is a description of the basic terminologies associated with options.

1. *Call options*: A contract where the buyer has the right to buy but not the obligation to buy. [14]
2. *Put Options*: A contract where the seller has the right to sell but not the obligation. [14]
3. *Exercise Price*: The price at which the holder can buy or sell the underlying stock; sometimes also referred to as the strike price. [15]
4. *Strike Time*: The future time at which the purchase is exercised. [12]

5. *Expiration date*: The date on or before which the holder can buy or sell the underlying stock. [15]
6. *American Option*: option can be purchased on or before the strike time. [12]
7. *European Option*: Option must be purchased on the strike time. [12]
8. *Payoff to a Contract*: the value of the option at expiration. [14]
9. *Underlying asset*: The asset or commodity on which the forward contract is based. [14]

Below I used our case study as an example to explain the above terminologies. The case study reads as:

A very eccentric , wealthy client and avid investor by the name of Emery Bowlander is interested in purchasing a European call option that provides him with the right to purchase shares of Fellare stock for \$44.00 on the first of February-12 weeks from today.

In here the option is a European call option. That means Mr. Bowlander has the right, but not the obligation to buy the stock on February first (which will be the expiration date). The 12 week time period will be the strike time and \$44 will be the exercise price. And obviously the underlying asset will be the Fellare stock. The payoff to the contract is the value of the option, which can be calculated by taking the difference between strike price (\$44) and the price on February 1.

3. Modeling the Stock Market Movement

3.1. Introduction

Unlike the model discrepancies we have today, in many instances a mathematical model can be used to represent a situation in engineering, social sciences, finance and in many of the interdisciplinary areas. Even though it may not be an exact representation of the stock market, a mathematical model can also be used to study the movement of the stock market to some extent. Keeping in mind the uncertainties on the stock price, the stock market movement can be modeled using probabilistic models. The most common and widely used models are:

- ✦ Lognormal Model
- ✦ Binomial Model
- ✦ Difference Model
- ✦ Geometric Brownian Model

In this part of the project I will restate the case study by incorporating the parameters provided on the case study, present an overview of Intel Corporation, and introduce a risk analysis software Oracle Crystal Ball Fusion edition. And at last a discussion of the above probabilistic models will be made using Ficare and Intel stocks as an example.

3.2. The Case Study Restated : Fellare Stock

Emer Bowlander is an investor interested in purchasing a European call option that provides him with the right to purchase shares of Fellare stock for \$44.00 on the first of February, 12 weeks from today. On February-1 if the price of the Fellare stock is above his exercise price of \$44.00, he will exercise the option and the value of the option will be the difference between the stock price and the exercise price. Otherwise he will not exercise the option, leaving the value of the option to be \$0. Having these two choices, the process of forecasting the option price will entirely depend on the price of the stock on February-1.

The value of the stock on February-1 is uncertain and its distribution can be modeled by using a lognormal distribution. The current annual interest rate (r) is 8 percent compounded continuously. The weekly interest rate (W) can be calculated as

$$W = (1 + r)^{\frac{1}{52}} - 1 = 0.148 \text{ percent}$$

The historical annual volatility (the standard deviation σ) is 30% and stock price for the current week is \$42.00. The historical weekly stock volatility (σ_w) can be calculated as

$$\sigma_w = \frac{\sigma}{\sqrt{52}} = 0.041603$$

The mean (drift parameter μ) can be calculated using

$$\mu = W - 0.5(\sigma_w)^2 = 0.000522$$

To get a better analysis on the option price, a modification on the Strike Price in the case study was made. The modified case study will have a Strike Price of \$43.00 (instead of \$44.00) but the other parameters were considered as they are presented on the case study.

3.3. Overview of Intel Corporation

In this study the Intel Corporation stock price will be studied as a means of validating the simulation models that will be developed. Before we use the stock price of Intel Corporation in our models, a brief overview of the Intel Corporation is presented below.

Based in Santa Clara, California and incorporated in 1968, Intel Corporation is an international supplier of integrated Circuits for computing and communications industries. [21] Microprocessor products offered by Intel in mobile and desktop computers, enterprise servers, and workstations include the following:

- Multi-core processors
- Quad-core processors
- 32 and 64 bit Architecture Microprocessors

Embedded designs are offered as well and are used in industrial equipment, point-of-sale systems, panel PCs, automotive information/entertainment systems, and medical equipment. The company also provides chipset products that interface input, output, and storage devices with the microprocessor, motherboards with connections for adding devices to the bus, flash memory products, networking products (connectivity and storage), and communications infrastructure products among others. Customers include equipment manufacturers design manufacturers, PC and communications products users, and others. Customers include equipment manufacturers, design manufactures, PC and communications products users, and others. Being such a giant corporation, the Intel has a well-known stock market history throughout the world. The stock Ticker for Intel Corporation is INTC. [21] In our model the historical prices will be used for analyzing the expected price of the stock and in determining the call option.

3.4. Oracle Crystal Ball Fusion Edition

The risk analysis software tool that I will be using in this thesis Project is Oracle Crystal Ball Fusion Edition. Oracle Crystal Ball (CB) is the leading spreadsheet-based application suite for predictive modeling, forecasting, simulation, and optimization. [4] It is an analytical tool that helps executives, analysts, and others make decisions by performing simulations on spreadsheet models. [4] The basic process for using CB is divided into three parts, as indicated below.

- ✦ Building a spreadsheet model that describes an uncertain situation.
- ✦ Run a simulation on it.
- ✦ Analyze the result.

Building a spreadsheet model and running a simulation on a set of data will enable us to observe the effect of certain variables on the expected output. The simulation on a spreadsheet model can be implemented using the well known simulation technique called Monte Carlo Simulation (MCS). MCS is a simulation model which randomly generates values for uncertain variables over and over to simulate a model. [4] This simulation is available as part of the CB package and will be the preferred way of running the simulations in this study. And a detailed discussion of the model is presented in Section 4.5.

3.5. Lognormal Model

Lognormal model represents a random variable whose logarithm has normal distribution. The lognormal distribution is the probability distribution that arises from the assumption that continuously compounded returns on the stock are normally distributed. [14] Mathematically speaking, a random variable Y follows a lognormal distribution if $\log(Y)$ is normally distributed

with mean μ and variance σ^2 . The mean μ of the lognormal distribution is called the *drift parameter* and the standard deviation σ is called the *volatility* parameter. The drift and volatility parameter are required in the process of estimating the price of the stock.

Assuming that the stock price changes over a short period of time and these prices are independent and identically distributed lognormal variables, the price of the stock (S) after t period of time can be calculated as

$$S = (S_0)e^{(\mu t \pm \sigma z\sqrt{t})} \quad (3.1)$$

where S_0 is the current price of the stock and z is the standard normal variable with $\mu = 0$ and $\sigma = 1$.

The above formula was implemented in Microsoft Excel spreadsheet and a simulated model was developed with the help of the risk analysis software, Crystal Ball. The process of building this model and analysis on the expected outcome is discussed below using the Ficare and Intel Corporation stocks as an example.

3.5.1. Using Lognormal Distribution for the Case Study

In this section we will attempt to build a model to forecast the price of the Ficare's stock with a strike period of twelve weeks. I divided the process of building and running the model using Microsoft Excel and CB into the following four steps.

1. Generating Spreadsheet Model in Excel
2. Defining Assumption Cell
3. Defining Forecast Cell

4. Running the Simulation and Analyzing the Result.

1. **Generating a Spreadsheet Model in Excel**

In generating the spreadsheet model, the first task will be identifying the parameters required for the model building process. Once the parameters are identified, two columns will be defined. The first column will be used to define the cell name associated with each parameter and the second one will be for their corresponding values. The parameters (with their corresponding cell name) required are the Current Price of the Stock, Strike Time, Drift and Volatility Parameters, Z-Value and Expected Stock Price. And the values associated with each parameter can be obtained either from the case study or using a formula or through an assumption. The values for Current Price of the Stock and Strike Time were directly obtained from the case study. The Drift and Volatility parameters were obtained from the calculation made on Page 8. To determine the values associated with the cell name Z-Value an assumption (an assumption that stock price follows a normal distribution with mean 0 and standard deviation 1) is required. In the spreadsheet developed, the Z-Value was generated using the Random Number Generation (RNG) data analysis tool, available in the data analysis tab of Excel 2010. When using RNG, a normal distribution with mean 0 and standard deviation 1 was selected as shown below in Figure 3.1.

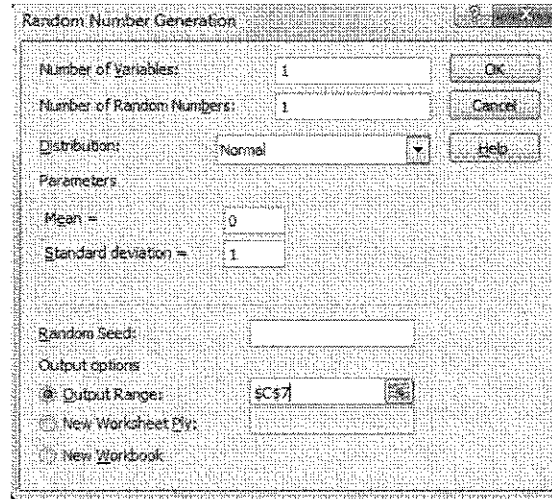


Figure 3.1 Screenshot of the Excel RNG data analysis tool window indicating a normal distribution with mean 0 and standard deviation 1.

The RNG generated the random number 0.0298. Once we have the above values, equation 3.1 can be used to determine the value associated with the cell name Expected Stock Price as shown below. This formula was implemented into our spreadsheet model. Hand calculation verified the rightness of the spreadsheet result as shown below.

$$\begin{aligned}
 S &= (S_0)e^{(ut \pm \sigma z\sqrt{t})} \\
 &= \$42 * e^{(0.00052*12 + 0.041603*(0.0298)*\sqrt{12})} \\
 &= \$42.44
 \end{aligned}$$

Table 3.1 provides a summary of the values associated with the above parameters. Hence the construction of the model is completed and it is ready to be used for simulation with the CB tools.

| Cell Name | Value | Value/ Formula/ Assumption required? |
|----------------------------|---------|--|
| Current Price of the Stock | \$42 | Value required: Case Study |
| Strike Time | 12 | Value required: Case Study |
| Drift Parameter | 0.00052 | Value required: Calculated on Page -8 |
| Volatility Parameter | 0.04160 | Value required: Calculated on Page-8 |
| Z-Value | 0.0298 | Assumption required: Normal Distribution |
| Expected Stock Price | \$42.44 | Formula required: Equation 3.1 |

Table 3.1: A table indicating the cell name defined and value/formula / assumption requirements in generating the spreadsheet model for the case study.

2. Defining the Assumption Cell

After generating the spreadsheet model, the next step will be defining an assumption cell. Assumption cells are input cells that contain values that you are unsure of: the uncertain independent variables in the problem you are trying to solve.[4]

In our case the only assumption cell is the Z-Value Cell. The assumption with this cell is that it follows a normal probability distribution with mean 0 and standard deviation 1. So to define Z-value as an assumption cell using CB, we follow the steps indicated below.

1. Select the Z-Value Cell and click on the Define Assumption Tab
2. From the distribution gallery select normal distribution. And enter the values of the mean and standard deviation as 0 and 1 respectively.
3. Then to complete defining the assumption cell, click ok. This will define Z-Value Cell as an assumption cell and will generate random values that will be used for the simulation.

Notice that the color of this cell will be changed to green, indicating that it is defined as an Assumption Cell.

3. Defining the Forecast Cell

By using the assumption cell defined above (Z-Value) and the parametric values we have in Table 3.1, we can define the Forecast Cell. Forecast Cells are output cells that contain formulas that refer to one or more assumption and decision variable cells. [4] In our case study the cell named Expected Stock Price will be our Forecast Cell. After making sure that Equation 3.1 is implemented properly in the spreadsheet, this cell can be defined as a Forecast Cell using CB by performing the following steps.

1. Select the Expected Stock Price Cell and click on Define Forecast tab
2. The Define Forecast Window will appear. Under the Name box type the “Expected Stock Price” as the name of the cell. In the Units box type “\$”. And click ok.
3. Once you click on ok, Expected Stock Price cell is defined to be the Forecast Cell and the simulated stock price will be stored in this cell. The cell color will be changed to light blue confirming the defining process of the Forecast Cell.

4. Running the Simulation and Analyzing the Result

After defining the Assumption and Forecast cells, the next step will be running the simulation. To have a better simulation model, I used 100,000 as the Number of trials to run. That is the maximum number of trials that CB runs before it stops the simulation and also the number of random numbers that will be generated in our Assumption Cell (Z-Value).

Before running the simulation, I used the **Step** command to generate one set of values. This will verify that the values generated are as per the assumption and also track down if there is a calculation error on the forecast. The one set of values generated for the Z-value and Expected Stock Price were 0.10744 and \$42.92, respectively. It appears that 0.10744 is a random

number from a normal distribution and the calculation made to get the Expected Stock Price (\$42.92) is valid calculation as per Equation 3.1. Now we are ready to run the simulation.

After setting the number of trials to 100,000, click on the **Start** command and the simulation process will start. While the simulation is running, the CB control panels will pop-up indicating the progress on the simulation. Once the simulation is complete, we can use the CB control panel option **Analyze** to see the list of available charts and different analysis tool to analyze the result. In this study I will use the Assumption, Forecast, Sensitivity and Scatter Charts to discuss the result of the simulation. I will also use the Statistics and Percentile Tables (obtained from the Forecast Chart) to present a numeric analysis on the result.

✦ **Assumption Charts**

Since we are using only one assumption, there is nothing much to discuss except making sure that the chart generated is the shape of a normally distributed probability function (bell – shaped) as shown in Figure 3.2. From the statistics view I checked that the mean is zero and the standard deviation is one. And the interval (range) for the distribution was observed to be [-4.47, 4.22]. On the top- right side of the chart (Figure 3.2) of the 100,000 trials made only 99,780 are displayed. This means that the other values (220) will fall out of the range, (-3,3), as indicated on the chart but still will be on the interval [-4.47, 4.22].

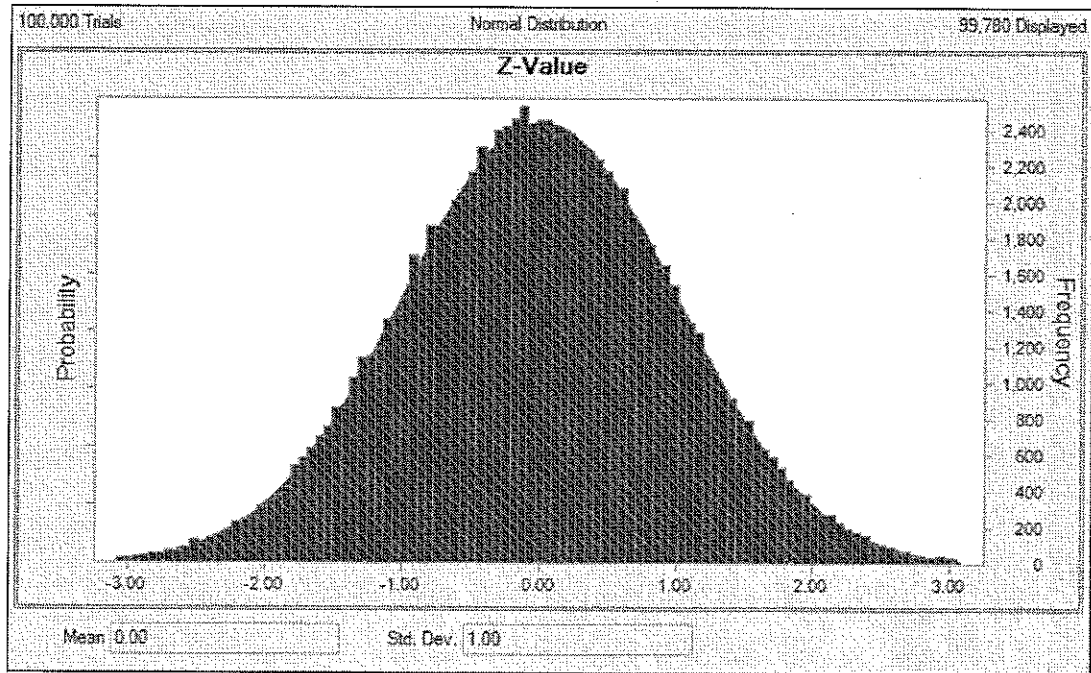


Figure 3.2 The Probability Assumption Chart for a Normal Distribution with mean 0 and standard deviation 1 simulated over 100,000 trials.

✦ Forecast Charts

This chart will enable us to analyze the simulation result in forecasting the Expected Stock Price. The chart indicated below (Figure 3.3) is the Frequency View of the Forecast Chart. Observe that the shape of the distribution is that of a normal distribution. The height of each bar indicates how many simulations resulted in the Expected Stock Price that fell in the corresponding interval. For example the tallest bar indicates that the stock prices were in between \$41 and \$42 for 4,500 trials. The probability scale to the left of the chart indicates the probability associated with the frequency. For example the highest value, 0.04, on the probability scale indicates that there is a 4% chance that 4000 trials of the simulation will fall within a certain range. The other quantity we have from the chart is the certainty level. The certainty level is the probability of achieving values within a certain range. For example, using the box at the bottom of the chart, the certainty level that the stock price will be in between \$41 and \$42 is

6.68%. By entering the range values or using the certainty grabbers (the small arrows on the left and right side), we can determine the certainty level for a specific value range. *The certainty level that the price of the stock is higher than the mean value (\$42.73) is around 47%.*

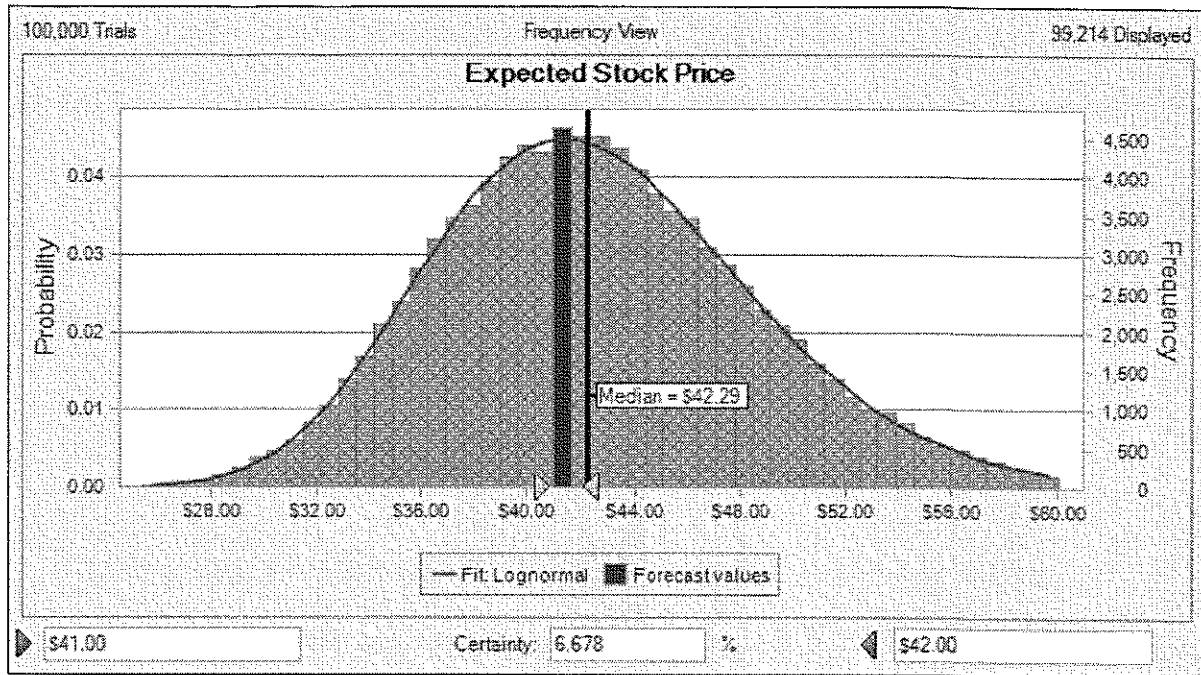


Figure 3.3 The Frequency View of the Forecast Chart on the Expected Stock Price indicating the frequency and probability of the possible ranges on the simulation.

To see a full set of descriptive statistics for the simulation, from the Forecast Window select **View** followed by **Statistics**. A discussion of the Statistics terms and the formula used in CB is presented below. These statistics will be referenced throughout the paper.

Assume that the results of the forecast cell after simulations are represented by $y_1, y_2, y_3, \dots, y_n$ where n is the number of iterations run before the simulation stops. Then the statistics we have in Table 3.2 can be described as follows.

- ✦ **Trials:** A three step process in which Oracle Crystal Ball, Fusion Edition generates random numbers for assumption cells, recalculates the spreadsheet model or models, and displays the results in a forecast chart. [4]

✦ Base Case: The Value in a Crystal Ball assumption, decision variable or forecast cell at the start of a simulation. [4]

✦ Mean : An arithmetic average of the forecast values and can be calculated as

$$\text{Mean} = \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i \quad [9]$$

✦ Median: The value midway (in terms of order) between the smallest possible value and the largest possible value. [4]

✦ Mode: The value which, if it exists, occurs most often. [4]

✦ Standard Deviation: a measure of dispersion, or spread, of a distribution from the mean. It is calculated as

$$\text{Standard Deviation} = s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2} \quad [9]$$

✦ Variance: the square of the standard deviation; i.e., the average of the squares of the deviations of a number of observations from their mean value. [4]

✦ Skewness: is a measure of asymmetry of a frequency distribution. The formula for skewness used by CB is

$$\text{Skewness} = \frac{1}{n} \sum_{i=1}^n \left(\frac{y_i - \bar{y}}{s} \right)^3 \quad [9]$$

✦ Kurtosis: is a measure of peakedness, which is equivalent to measuring tail thickness. The higher the kurtosis, the closer the points of the curve lie to the mode of the curve. A normal distribution curve has a kurtosis of 3. And it is calculated by the formula

$$\text{Kurtosis} = \frac{1}{n} \sum_{i=1}^n \left(\frac{y_i - \bar{y}}{s} \right)^4 \quad [9]$$

✦ Coefficient of Variability: a measure of relative variation that relates the standard deviation to the mean. [4]

- ✦ **Minimum and Maximum Value:** the smallest and the largest value of all the observed forecast values.
- ✦ **Mean Standard Error:** The standard deviation of the distribution of possible sample means. This statistic gives one indication of how accurate the simulation is. **[4]**
- ✦ **Correlation Coefficient:** A number between -1 and 1 that specifies mathematically the degree of positive or negative correlation between assumption cells. **[4]**

Based upon the above definitions and Table 3.2, the forecast statistics for Expected Stock Price can be summarized as follows:

With 100,000 trials the mean of the Expected Stock Price for Fellare Stock was about \$42.73 with a standard deviation of \$6.19. The median was found to be \$42.29. From the simulation run, the minimum stock price generated was \$21.78 and the maximum was \$77.85. The mean standard error which shows the precision of the simulation was about 0.02.

| Forecast: Expected Stock Price | |
|--------------------------------|-----------------|
| Statistic | Forecast values |
| Trials | 100,000 |
| Base Case | \$42.26 |
| Mean | \$42.73 |
| Median | \$42.29 |
| Mode | '--- |
| Standard Deviation | 6.19 |
| Variance | \$38.34 |
| Skewness | 0.4484 |
| Kurtosis | 3.37 |
| Coeff. of Variability | 0.1449 |
| Minimum | \$21.78 |
| Maximum | \$77.85 |
| Mean Std. Error | 0.02 |

Table 3.2: Statistic View of the Forecast showing numerical summaries of the Forecast for Fellare Stock

Another analysis tool that needs to be considered is the Percentile View of the Forecast. Percentile is the percent chance, or probability, of a forecast value being less than or equal to the value that corresponds to the percentile (the default). [4] The percentile information was displayed from the Forecast Window by choosing **View** and then **Percentile** as shown below in Table 3.3. The percentile was calculated with a 10% increment. For example the 60th percentile in Table 3.3 indicates that there is a 60% chance of a forecast value being equal to or less than \$43.82.

| Forecast: Expected Stock Price | |
|--------------------------------|-----------------|
| Percentile | Forecast values |
| 0% | \$21.78 |
| 10% | \$35.18 |
| 20% | \$37.44 |
| 30% | \$39.22 |
| 40% | \$40.79 |
| 50% | \$42.29 |
| 60% | \$43.82 |
| 70% | \$45.59 |
| 80% | \$47.73 |
| 90% | \$50.85 |
| 100% | \$77.85 |

Table 3.3: Percentile View of the Forecast calculated with a 10% increment.

✦ Sensitivity Charts

The next analysis will be in determining how much the assumption used above affects the forecast. The only assumption used is the Z-Value; hence we can expect a very high effect on the forecast. CB can generate a chart called Sensitivity Chart that shows the effect of the assumptions on the forecast. It shows the influence of each assumption cell on a particular forecast cell. To generate the sensitivity chart shown in Figure 3.4, from the forecast window click on the Forecast Option and then select Open Sensitivity Chart.

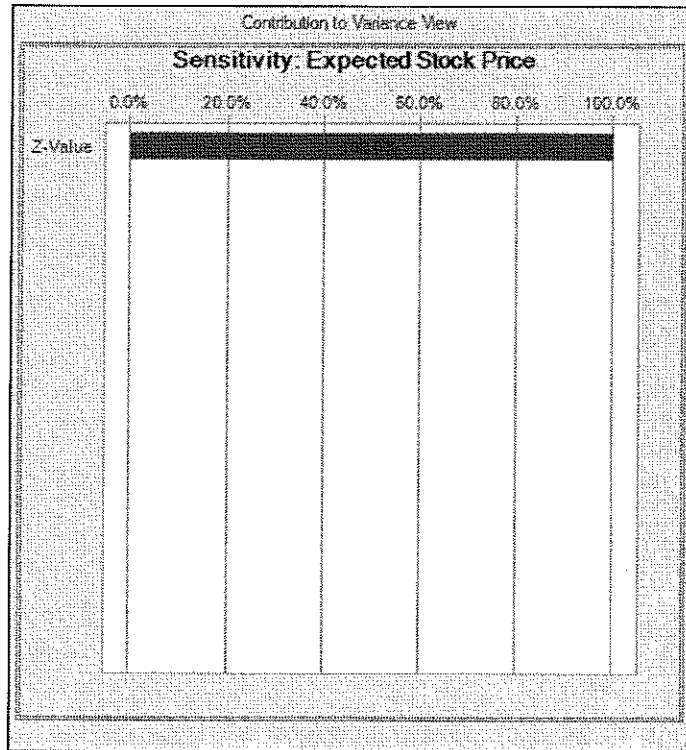


Figure 3.4: Sensitivity Chart for the Expected Stock Price.

As Z-Value is the only assumption we used, it accounts for 100% of the variance in the forecast values.

✦ Scatter Charts

Scatter Charts show correlations, dependencies, and other relationships between pairs of forecasts and assumptions plotted against each other. [4] Figure 3.5 shows the scatter chart for our case study. This chart indicates that there is a positive correlation between the Expected Stock Price and the Z-Value.

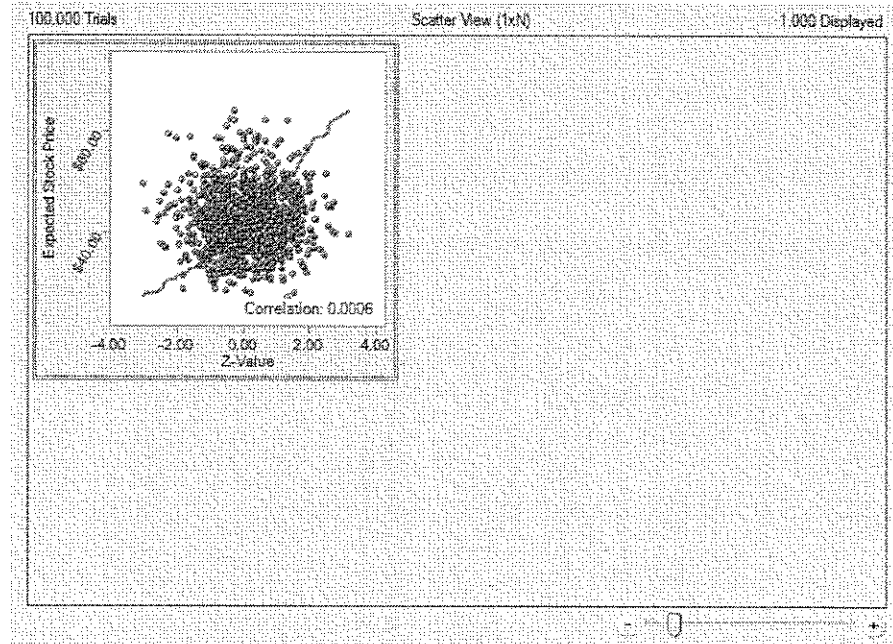


Figure 3.5 Scatter Chart for Expected Stock Price and Z-value for Fellare Stock

3.5.2. Using lognormal Model for Intel Corporation

To see the appropriateness of the lognormal model, the model was implemented on real stock price of Intel Corporation and the forecasted price was compared with the actual price on the market. In a similar way as it was done for Fellare stock, six parameters were identified. The first parameter, Current Price of the Stock dated on 02-22-2011, was obtained from Yahoo Finance. [20] The recorded price on this date was \$21.69. Again in here a Strike Time of twelve weeks will be used. Hence the Expected Stock Price will be on 05-11-2011 (twelve weeks from 02-22-2011). The Drift and Volatility parameters were obtained by considering the two year weekly historical stock prices of the INTC dated 11-10-2008 to 11-8-2010, downloaded from yahoo finance (with 105 total number of stock prices (n)). [20] By assuming that the stock prices follow a lognormal distribution, I took the natural logarithm of the ratio of $S(n+1)$ to $S(n)$ to determine the return value of the stock. The average of the 105 return values will determine the weekly Drift Parameter and standard deviation of the returns determines the weekly Volatility

Parameter. For example on 11/8/2010 the stock price was 20.89 and on 11/1/2010 it was 20.9.

By taking the natural logarithm of the ratio of 20.89 to 20.9 $\ln\left(\frac{20.89}{20.9}\right)$ will yield a return value of -0.00048. Following similar procedure, excel spreadsheet was used to generate the return values on the intended date range as shown below. The weekly drift parameter was calculated as

$$Drift = \frac{\sum_1^n \left(\frac{S(n+1)}{S(n)}\right)}{n} = 0.00510$$

And the weekly volatility was calculated using the formula

$$Volatility = \sqrt{\frac{1}{n-1} \sum_1^n (S(n) - Drift)^2} = 0.043813$$

These values will be used and referenced throughout this paper.

| Date | Stock price | Return |
|------------|--------------------------|-----------------|
| 11/8/2010 | 20.89 | -0.00048 |
| 11/1/2010 | 20.9 | 0.065751 |
| 10/25/2010 | 19.57 | 0.010272 |
| 10/18/2010 | 19.37 | 0.026682 |
| . | . | . |
| . | . | . |
| . | . | . |
| 12/8/2008 | 13.61 | 0.103648 |
| 12/1/2008 | 12.27 | -0.03759 |
| 11/24/2008 | 12.74 | 0.051541 |
| 11/17/2008 | 12.1 | -0.01558 |
| 11/10/2008 | 12.29 | |
| | Weekly drift | 0.005101 |
| | Weekly Volatility | 0.043813 |

Table 3.4: Historical stock prices of Intel Corporation 11\10\2008-11\8\2010 indicating the calculated drift and volatility parameters

Following the same procedure as it was done for Fellare Stock (Page 11-12), the excel spreadsheet model was developed. Again in here our Assumption Cell will be the Z-Value Cell and our Forecast Cell is the Expected Stock Price cell. The same assumption (that is normal distribution with a mean of 0 and standard deviation of one) was used for the Z-Value cell. The formula implemented in the Expected Value cell was again Equation 3.1. The only difference in the spread sheet model was the values for the Current Price of the Stock, Drift and Volatility Parameters. The values associated with the above six parameters are summarized in Table 3.5. From Yahoo Finance the actual price of the stock on 05-16-2011 was recorded to be \$23.22. [20] This price will be used to check how accurate our Forecasted price is.

| Cell Name | Value | Value/ Formula/ Assumption required? |
|----------------------------|---------|--|
| Current Price of the Stock | \$21.69 | Value required: Yahoo Finance |
| Strike Time | 12 | Value required |
| Drift Parameter | 0.00510 | Value required: Calculated |
| Volatility Parameter | 0.04381 | Value required: Calculated |
| Z-Value | 0.432 | Assumption required: Normal Distribution |
| Expected Stock Price | \$24.62 | Formula required: Equation 3.1 |

Table 3.5: The cell names, the value, formula and assumptions used in generating the spreadsheet model for Intel Corporation

After generating the spreadsheet model, with 100,000 numbers of trials, the simulation was processed following the same steps as in page 14. A brief discussion of the simulation analysis is presented below.

For presentation purpose, a split view of Frequency Chart, Statistics and Percentile for the Expected Stock Price were generated all in one as shown in Figure 3.6.

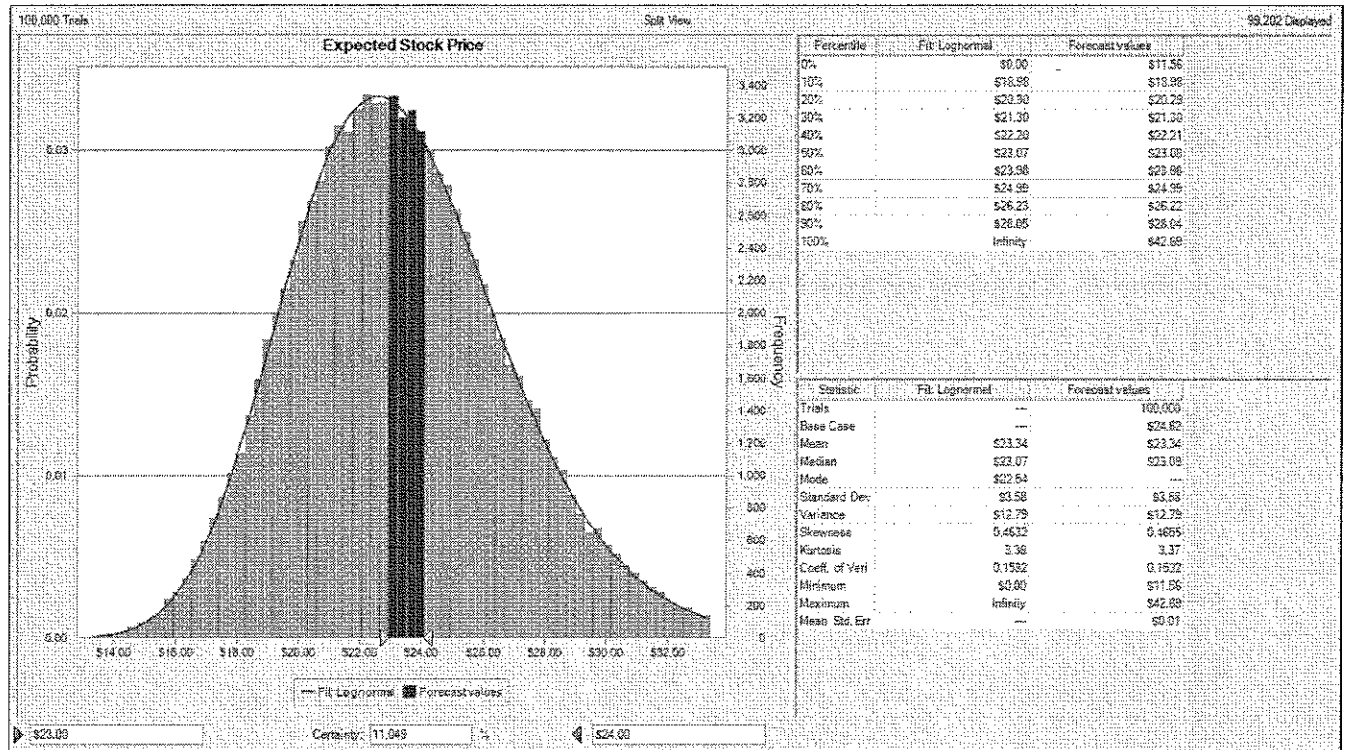


Figure 3.6: Split View of the Frequency View Chart (left side), Percentile (right top) and Statistics (right bottom) for the Expected Stock Price.

From the frequency chart observe that the certainty level that the stock price will be in between \$23 and \$24 is 11.049%. And the certainty level that the Expected Stock price is greater than the mean (\$23.34) is around 47.1%.

From the statistics view, the mean and the standard deviation of the Expected Stock Price for the 100,000 trials were observed to be \$23.34 and \$3.58, respectively. The minimum stock price was \$11.56 and the maximum was \$77.85. The mean standard error was 0.01.

From the percentile view, it is observed that there is a 60% chance for the Expected Stock Price to be less than \$23.98. As we only have one assumption, 100% of the variance in the forecast value is due to this assumption (Z-Value). This can be checked from the sensitivity chart shown in Figure 3.7.

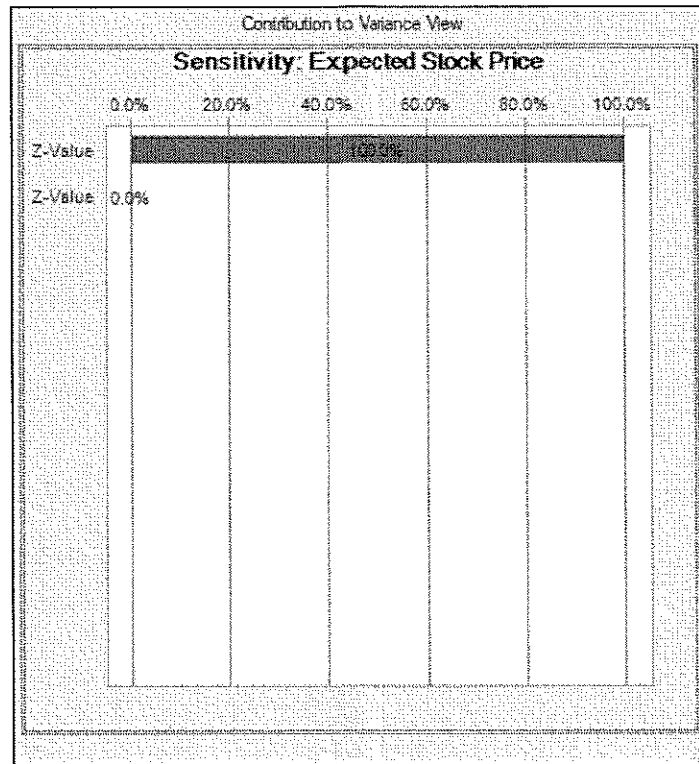


Figure 3.7: Sensitivity Chart for the Expected Stock Price

From the point of view of scatter chart there is a positive correlation between the values generated for the Expected Stock price and the Z-Value as shown in Figure 3.8.

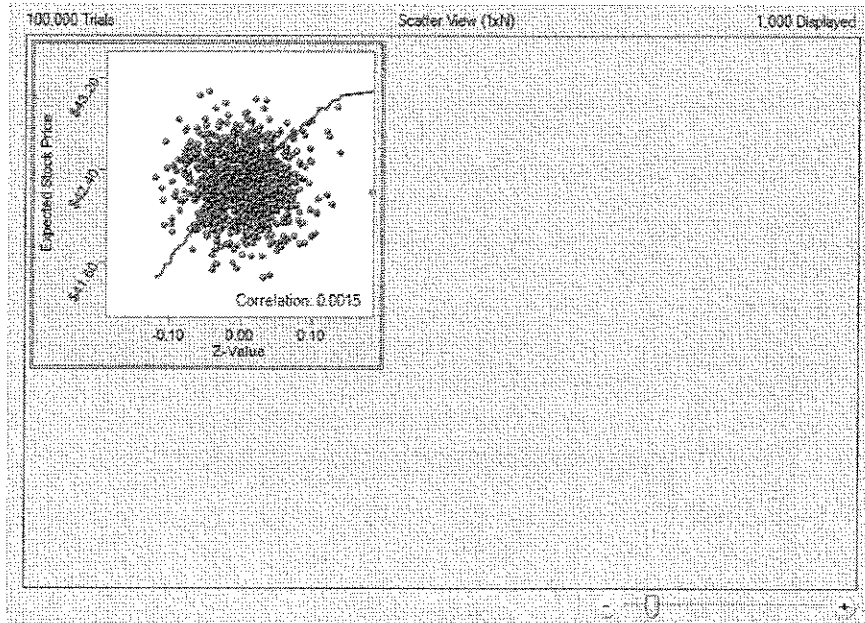


Figure 3.8: Scatter Chart for Expected Stock Price and Z-Value for Intel Corporation

Our simulation result indicated that the mean value of the Expected Price of the Stock to be \$23.34. When compared to the actual price obtained from Yahoo Finance (the value was about \$23.22), there is a deviation of about \$0.12 from the forecasted price.

3.6. The Binomial Model

The Binomial model is one of the mathematical models built based on the assumption that after a certain period of time the stock price will either go up or down. That means to generate the model, we need to assume that there are only two possible states: up or down. The probability that it will go up or down can be restated in terms of success or failure on the possible outcome. Let's denote the probability of success (the probability that the price will go up) by p and the probability of failure (the probability that the price will go down) by $q = 1 - p$. These probabilities can be computed by taking n independent events of Bernoulli trials where each trial has two possible outcomes: success or failure. A random variable Y is said to have a binomial distribution based on n trials with total number of successes y and probability of success p if Y has the following probability function [3]

$$p(y) = \binom{n}{y} p^y q^{n-y}, y = 0, 1, 2, \dots, n \text{ and } 0 \leq p \leq 1 \quad (3.2)$$

To estimate the price of the stock over a short period of time:

- ✦ We assume that the movement of the stock price follows a binomial distribution as defined above.
 - ✦ Consider the fact that successive movements in a binomial distribution are independent.
 - ✦ And the following parameters are predefined.
1. The present price of the stock is denoted as S_0
 2. After n periods the price of the stock is denoted as $S(n)$
 3. The factor by which the price goes up is denoted by u .
 4. The factor by which the price goes down is denoted by d

5. The probability that the price will go up is denoted by p and the probability that it will go down is denoted by $1 - p$.

In n periods the stock's price can go up x times and go down $(n - x)$ times, therefore

$$S(n, x) = u^x d^{(n-x)} S_0 \quad (3.3)$$

at probability $\binom{n}{x} p^x (1 - p)^{n-x}$. [12] After determining the up and down factors, this formula can be used to estimate the stock price for n periods. And the Expected Price of the Stock can be estimated using the combinational analysis formula

$$\sum_{x=0}^n \binom{n}{x} p^x (1 - p)^{n-x} S(n, x) \quad (3.4)$$

$S(n, x)$ is the price of the stock at period n associated with the number of times (x) the price goes up.

Now let's see how we can calculate the factor by which the stock price will either go up or down. As there is always uncertainty associated with stock price, we will not have a definite up or down value on the stock. But still we can measure the uncertainty about the stock return by considering the annual standard deviation of the stock, the deviation of the actual price of the stock from the drift (the mean estimate of the historical prices). This annualized standard deviation σ is called the historical annual stock volatility. To do this estimation, consider the case where we have an infinite number of periods (that is, $n \rightarrow \infty$). It seems that the binomial distribution can be approximated by lognormal distribution. Based on this approximation, the drift and volatility parameters can be calculated as: [12]

$$\text{Drift Parameter } \mu = p \log u + (1 - p) \log d$$

$$\text{Volatility Parameter } \sigma = \sqrt{p(1-p)} \left(\log \frac{u}{d} \right)$$

Assuming that the stock will not provide a dividend yield and using σ over a period of time t with annual interest rate r , we can calculate u and d as follows: [14]

$$u = e^{(rt + \sigma\sqrt{t})} \quad (3.5)$$

$$d = e^{(rt - \sigma\sqrt{t})} \quad (3.6)$$

The probability that the price will go up (p) can be calculated as: [14]

$$p = \frac{e^{rt} - d}{u - d} \quad (3.7)$$

This probability is called the risk neutral probability, which will be discussed later in Section 4.3. I will illustrate the implementation of the binomial model using the Fellare Stock and check the validity of the model using real stock prices on Intel Corporation.

3.6.1. Using Binomial Model for Fellare Stock

To build a more realistic binomial model, I divided the time to expiration into 12 periods. One period indicates a one week interval (one week). The table below specifies the weekly values we have for Weekly Interest Rate (r), *Strike Time* (t) and *Weekly Historical Volatility* (σ) for Fellare Stock in our case study. The values for r and σ were obtained from the calculations made on Page-8.

| | |
|--|----------|
| Weekly Interest Rate | 0.00148 |
| Strike time in weeks(t) | 1 |
| Weekly Historical Volatility(σ_w) | 0.041603 |

Table 3.6: The Weekly Interest Rate, Strike Time and Volatility for the Fellare's Stock

Inserting these values into equation (3.5) and (3.6), the up (u) and down (d) values will turn out to be 1.044 and 0.9607. And the probability that the price will go up (p) is 0.4896 (using equation 3.7).

With an initial stock price (S_0) of \$42 and the above u and d values, we can forecast the expected price of the stock for each binomial period. A simplified way of calculating this price will be constructing a binomial tree. Below is a discussion of the construction of the binomial tree and the stock pricing process.

One- Period Binomial Tree

Assume the starting price of the stock to be S_0 . If the price happens to go up, the up-value can be estimated as uS_0 with a probability of $p = 0.4896$. Otherwise the price will have a down-value of dS_0 with a probability of $1 - p = 0.5104$. The result of the calculation is indicated in Table 3.7 below.

| | | |
|------------|----|-------|
| Up-Value | \$ | 43.85 |
| Down-Value | \$ | 40.35 |

Table 3.7: The estimated up and Down prices for Fellare's Stock after one week

Figure 3.9 shows the one-period binomial tree with its up and down prices. This tree was constructed using Mathematica. The source codes used in constructing the tree were obtained

from the Wolfram Demonstration Project titled Binomial Tree. [7] The Expected Stock Price after one-period ($n=1$) can be calculated using Equation 3.4 as indicated below.

$$\sum_{x=0}^1 (C(1, x) * p^{1-x} * p^x) * S(1, x) = \$42.06$$

Hence the Expected Stock Price after one-period is \$42.06

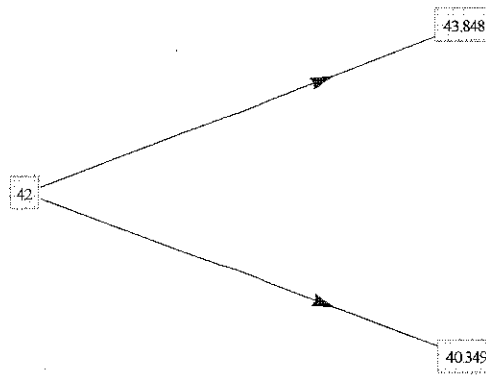


Figure 3.9: One-Period Binomial Tree for Fellare Stock implemented in Mathematica using the source code Binomial Tree from Wolfram Demonstration Project.

Two-Period Binomial Tree

The construction of the two- period binomial tree will be based on what we have on one- period binomial tree. To do so we follow the same trend except that our starting values will be $uS_0 = \$43.85$ and $dS_d = \$40.35$ as calculated above. The calculation is presented below by considering two cases.

Case 1: Assume the price in one binomial tree was *up*: \$43.85

In this case the next outcome can be either up or down. If it is up, we have $n = 2$ and $X = 2$ and using equation (3.3), the stock price will be

$$\begin{aligned} S(n) &= u^2 S_0 \\ &= 45.78 \end{aligned}$$

with a probability of $p^2 = 0.2397$

If it is down, we have $n = 2$ and $X = 1$ and again using equation (3.3), the stock price will be

$$\begin{aligned} S(n) &= u^2 d S_0 \\ &= 42.12 \text{ with a probability of } p(1-p) = 0.2499. \end{aligned}$$

Case 2: Assume that the price in one binomial tree was down: \$41.50

Again in here we have two possible outcomes: up or down. If it is up (we have $n=2$ and $X=1$), the stock price will be

$$\begin{aligned} S(n) &= u d S_0 \\ &= \$42.12 \end{aligned}$$

with a probability of $(1-p)p = 0.24999$.

If it's down, we have $n=2$ and $X=0$ and using equation (3.3), the stock price will be

$$\begin{aligned} S(n) &= d^2 S_0 \\ &= \$38.76 \end{aligned}$$

with a probability of $(1-p)^2 = 0.2605$

Again, the two- period binomial tree can be constructed using the Binomial Tree source code from Wolfram Demonstration Project implemented in Mathematica. [7]

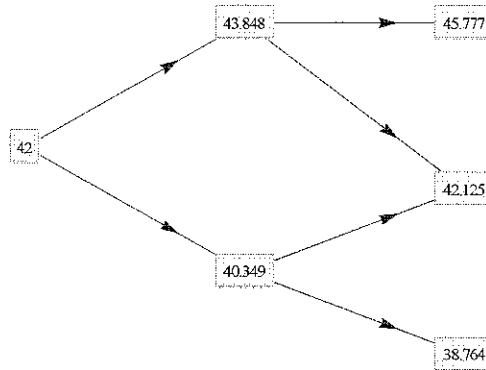


Figure 3.10: Two-Period Binomial Tree for Ficare Stock implemented in Mathematica using the source code *Binomial Tree* from Wolfram Demonstration Project.

Again, the Expected Stock Price can be calculated using Equation 3.4 as shown below. Observe that $S(2,0) = \$45.77, S(2,1) = \42.12 and $S(2,2) = \$38.764$

$$\sum_{x=0}^2 (C(2, x) * p^{1-x} * p^x) * S(2, x) = \$42.13$$

Hence the Expected Stock Price after two-binomial period is \$42.13.

Following similar procedures, one can construct the n-period binomial tree and also estimate the stock price after n periods. Below are examples of three and four period binomial trees using the source codes from Wolfram Demonstration Project implemented in Mathematica.[7]

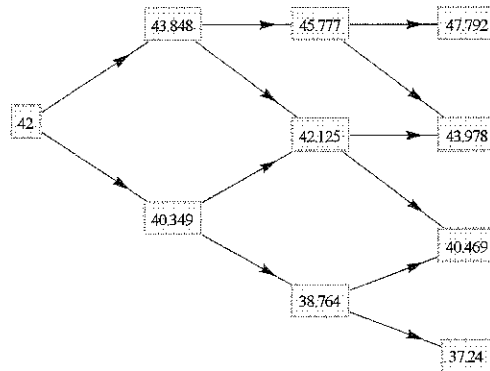


Figure 3.11: Three-Period Binomial Tree for Fellare Stock implemented in Mathematica using the source code Binomial Tree from Wolfram Demonstration Project.

The Expected Stock Price after three-binomial period is calculated using Equation 3.4 as shown below. Observe that $S(3,3) = \$47.92$, $S(3,2) = \$43.98$, $S(3,1) = \$40.47$ and

$$S(3,0) = \$37.24$$

$$\sum_{x=0}^3 (C(3,x) * p^{1-x} * p^x) * S(3,x) = \$42.19$$

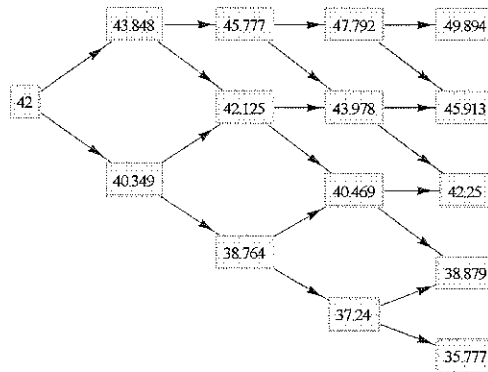


Figure 3.12: Four-Period Binomial Tree for Fellare Stock implemented in Mathematica using the source code Binomial Tree from Wolfram Demonstration Project.

The Expected Stock Price after four-binomial period can also be calculated using Equation 3.4 as shown below. Observe in here that $S(4,4) = \$49.89$, $S(4,3) = \$45.91$, $S(4,2) = \$42.25$, $S(4,1) = \$38.88$ and $S(4,0) = \$35.78$

$$\sum_{x=0}^4 (C(4, x) * p^{1-x} * p^x) * S(4, x) = \$41.88$$

Following the same routine as above, one can determine the expected values of the stock at the end of n-binomial period by generating n-period binomial tree. But this process will be a very tedious and a long process if done either by hand or using Mathematica (long source code). A better way (not the best) to do this is using Microsoft Excel Spreadsheet. A Twelve-Period Binomial Model incorporating eight parameters (listed in Table 3.8) was generated. The values associated with each parameter are indicated in Table 3.8. The last column in this table lists the method used to obtain the values.

| Parameter Name | Value | Method Obtained |
|-----------------------------------|----------|----------------------------|
| Current Price of the Stock | \$42.00 | Case Study |
| Weekly Interest Rate | 0.148% | Calculation made on Page-8 |
| Strike Time in Weeks (One-period) | 1 | Case Study |
| Weekly Historical Volatility | 0.041603 | Calculation made on Page-8 |
| UP-Value Factor | 1.04402 | Using Equation 3.5 |
| DOWN-Value Factor | 0.96067 | Using Equation 3.6 |
| Up Probability | 0.48960 | Using Equation 3.7 |
| Expected Stock Price (Week -12) | \$42.75 | Using Equation 3.4 |

Table 3.8: List of Parameters Name, Value and Method Obtained for Fellare Stock

Once the above parameters and their corresponding values are obtained, the Expected Stock Price on each period can be calculated using Excel Spreadsheet. For example, to calculate the one period Expected Stock Price using Excel Spreadsheet, we define four cells and perform the following calculations.

- ✦ The first cell will be used to store the Current Price of the Stock (\$42.00).
- ✦ The second cell will be used to store the Up-Value of the stock. To do so multiply the cell that contains the Up-Value Factor (1.04402) by the cell that contains Current Price of the Stock.
- ✦ The third cell will be used to store the Down-Value of the stock. In here also multiply the cell that contains the Down-Value Factor (0.96067) by the cell that contains the Current Price of the Stock.
- ✦ The Fourth cell will be used to store the estimated Expected Price of the Stock. To do so, implement Equation 3.4 in this cell.

Figure 3.12 shows the result of the above four calculations.

| Period(t) | 0 | 1 |
|-------------|----|----------|
| | | \$ 43.85 |
| \$ 42.00 | | \$ 40.35 |
| S(t) | 42 | \$ 42.06 |

Figure 3.13: One-Period Binomial Tree Using Microsoft Excel for Fellare Stock.

The Current price of the stock ($S(0)$) is \$42, the Up-Value of the stock is \$43.85 and the Down-Value of the stock is \$40.35. The one-period Expected Stock Price ($S(1)$) is \$42.06.

Performing similar calculation again and again, the twelve-period binomial tree model can be developed as shown in Figure 3.13 below. Hence using the Binomial Method of Pricing Stock, the Expected Stock Price for Fellare stock at the end of twelve weeks is \$42.75.

| Period(t) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|-----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| | | | | | | | | | | | | \$ 67.46 | \$ 70.43 |
| | | | | | | | | | | \$ 64.62 | \$ 62.08 | \$ 64.81 | |
| | | | | | | | | \$ 59.28 | \$ 56.95 | \$ 59.46 | \$ 57.12 | \$ 59.64 | |
| | | | | | | \$ 54.39 | \$ 52.25 | \$ 54.55 | \$ 52.41 | \$ 50.34 | \$ 52.56 | \$ 54.87 | |
| | | | \$ 49.90 | \$ 47.94 | \$ 47.94 | \$ 50.05 | \$ 48.08 | \$ 50.20 | \$ 48.22 | \$ 46.32 | \$ 48.36 | \$ 50.49 | |
| | | \$ 45.78 | \$ 45.92 | \$ 44.11 | \$ 46.05 | \$ 46.19 | \$ 44.37 | \$ 46.32 | \$ 44.50 | \$ 42.63 | \$ 44.66 | \$ 46.46 | |
| \$ 42.00 | \$ 43.85 | \$ 42.12 | \$ 43.98 | \$ 42.25 | \$ 44.37 | \$ 42.50 | \$ 44.37 | \$ 42.63 | \$ 44.50 | \$ 42.75 | \$ 44.75 | \$ 42.75 | |
| | \$ 40.35 | \$ 40.47 | \$ 38.88 | \$ 40.59 | \$ 38.99 | \$ 40.71 | \$ 39.11 | \$ 40.83 | \$ 39.22 | \$ 40.95 | \$ 39.34 | \$ 39.34 | |
| | | \$ 38.76 | \$ 37.24 | \$ 38.88 | \$ 37.35 | \$ 35.88 | \$ 37.46 | \$ 35.98 | \$ 36.09 | \$ 36.20 | \$ 36.20 | \$ 36.20 | |
| | | | \$ 35.77 | \$ 34.37 | \$ 33.01 | \$ 34.47 | \$ 33.11 | \$ 34.57 | \$ 33.21 | \$ 34.67 | \$ 33.31 | \$ 33.31 | |
| | | | | | | \$ 31.72 | \$ 31.11 | \$ 31.81 | \$ 31.90 | \$ 31.90 | \$ 31.90 | \$ 31.90 | |
| | | | | | | | \$ 30.47 | \$ 30.56 | \$ 30.65 | \$ 30.65 | \$ 30.65 | \$ 30.65 | |
| | | | | | | | | \$ 29.27 | \$ 29.36 | \$ 29.36 | \$ 29.36 | \$ 29.36 | |
| | | | | | | | | | \$ 28.12 | \$ 28.20 | \$ 28.20 | \$ 28.20 | |
| | | | | | | | | | | \$ 27.01 | \$ 27.01 | \$ 27.01 | |
| | | | | | | | | | | | | \$ 25.95 | |
| S(t) | \$ 42.00 | \$ 42.06 | \$ 42.12 | \$ 42.19 | \$ 41.88 | \$ 42.31 | \$ 42.37 | \$ 42.44 | \$ 42.50 | \$ 42.56 | \$ 42.63 | \$ 42.69 | \$ 42.75 |

Figure 3.14: Excel Spreadsheet implementation of the Binomial Method of Stock Pricing on Fellare stock. The price of the stock on week one, $S(0)$, was \$42 and after twelve periods (12 weeks) the price $S(12)$ is \$42.75 as indicated on the last row of the figure.

3.6.2. Using Binomial Model for Intel Corporation

To check the appropriateness of the Binomial model, I used the model for a real stock price on the market: Intel Corporation stock. To use the model, we need to identify eight parameters and their associated values. Below is a brief description of the eight parameters listed in Table 3.9. The Interest rate is obtained from the US Department of Treasury. The daily Treasury Bill Rate over 5 years was determined to be 1.6%. [18] The Weekly Interest Rate can be calculated by considering the number of weeks we might have within five years as shown below.

$$r = (1 + 1.6\%)^{\frac{1}{5 \times 52}} = 0.0320\%$$

The Weekly Historical Volatility can be determined by taking into account of the two-year weekly historical stock prices (11\10\2008 – 11\8\2010) of the Intel Corporation downloaded from Yahoo Finance. [20] This calculation was done on Page-23 and a value of 0.043813 was obtained for the Weekly Historical Volatility. The Strike Time will be a period (one- week).

| Parameter Name | Value | Method Obtained |
|--|---------|-----------------------------|
| Current Price of the Stock (on 02-22-10) | \$21.69 | Yahoo Finance |
| Weekly Interest Rate | 0.032% | Calculation made on Page-32 |
| Strike Time in Weeks (One-period) | 1 | Taken form the case study |
| Weekly Historical Volatility | 0.04381 | Calculation made on Page-8 |
| UP-Value Factor | 1.01276 | Using Equation 3.5 |
| DOWN-Value Factor | 0.98746 | Using Equation 3.6 |
| Up Probability | 0.49684 | Using Equation 3.7 |
| Expected Stock Price (Week -12) | \$21.70 | Using Equation 3.4 |

Table 3.9: List of Parameters Name, Value and Method Obtained for INTC Stock

Following a similar procedure as it was done for Fellare Stock, the twelve- period binomial tree was constructed using Microsoft Excel spreadsheet. The spreadsheet below indicates that the calculated expected price of the stock on 5\16\2011, using the binomial model, to be \$21.70. From yahoo finance, the exact value of the stock on 5\16\2011 was \$23.22, with \$1.52 deviation from the above forecasted price. McDonald [14] suggests that dividing the time to expiration into more periods will generate a more realistic binomial tree.

| Period(t) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
|-----------|----------|----------|----------|-------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| | | | | | | | | | | | | | \$ 25.25 | |
| | | | | | | | | | | | \$ 24.93 | | \$ 24.62 | |
| | | | | | | | | | | \$ 24.31 | | \$ 24.31 | \$ 24.62 | |
| | | | | | | | | \$ 24.00 | | \$ 24.01 | | \$ 24.01 | \$ 24.01 | |
| | | | | | | | \$ 23.70 | | \$ 23.70 | | \$ 23.70 | | \$ 23.70 | |
| | | | | | | \$ 23.41 | | \$ 23.41 | | \$ 23.41 | | \$ 23.41 | \$ 23.41 | |
| | | | | | \$ 23.11 | | \$ 23.11 | | \$ 23.11 | | \$ 23.11 | | \$ 23.11 | |
| | | | | 22.81798485 | | \$ 22.82 | | \$ 22.82 | | \$ 22.82 | | \$ 22.82 | \$ 22.82 | |
| | | | \$ 22.53 | | \$ 22.53 | | \$ 22.53 | | \$ 22.53 | | \$ 22.53 | | \$ 22.54 | |
| | | \$ 22.25 | | \$ 22.25 | | \$ 22.25 | | \$ 22.25 | | \$ 22.25 | | \$ 22.25 | \$ 22.25 | |
| | \$ 21.97 | | \$ 21.97 | | \$ 21.97 | | \$ 21.97 | | \$ 21.97 | | \$ 21.97 | | \$ 21.97 | |
| \$ 21.69 | | \$ 21.69 | | \$ 21.69 | | \$ 21.69 | | \$ 21.69 | | \$ 21.69 | | \$ 21.70 | \$ 21.70 | |
| | \$ 21.42 | | \$ 21.42 | | \$ 21.42 | | \$ 21.42 | | \$ 21.42 | | \$ 21.42 | | \$ 21.42 | |
| | | \$ 21.15 | | \$ 21.15 | | \$ 21.15 | | \$ 21.15 | | \$ 21.15 | | \$ 21.15 | \$ 21.16 | |
| | | | \$ 20.88 | | \$ 20.89 | | \$ 20.89 | | \$ 20.89 | | \$ 20.89 | | \$ 20.89 | |
| | | | | \$ 20.62 | | \$ 20.62 | | \$ 20.62 | | \$ 20.63 | | \$ 20.63 | \$ 20.63 | |
| | | | | | \$ 20.36 | | \$ 20.36 | | \$ 20.37 | | \$ 20.37 | | \$ 20.37 | |
| | | | | | | \$ 20.11 | | \$ 20.11 | | \$ 20.11 | | \$ 20.11 | \$ 20.11 | |
| | | | | | | | \$ 19.86 | | \$ 19.86 | | \$ 19.86 | | \$ 19.86 | |
| | | | | | | | | \$ 19.61 | | \$ 19.61 | | \$ 19.61 | \$ 19.61 | |
| | | | | | | | | | \$ 19.36 | | \$ 19.36 | | \$ 19.36 | |
| | | | | | | | | | | \$ 19.12 | | \$ 19.12 | \$ 19.12 | |
| | | | | | | | | | | | \$ 18.88 | | \$ 18.88 | |
| | | | | | | | | | | | | | \$ 18.64 | |
| S(t) | \$ 21.69 | \$ 21.69 | \$ 21.69 | \$ 21.69 | \$ 21.63 | \$ 21.69 | \$ 21.69 | \$ 21.69 | \$ 21.69 | \$ 21.69 | \$ 21.70 | \$ 21.70 | \$ 21.70 | \$ 21.70 |

Figure 3.15: Excel Spreadsheet implementation of the Binomial Method of Stock Pricing for Intel Corporation. The current price of the stock, $S(0)$, is \$21.69 and after twelve periods (12 weeks) the price $S(12)$ is \$21.70 as indicated on the last row of the figure.

3.7. The Difference Model

This model requires observing the price of the stock periodically. Let $S(n)$ is the price of the stock at period n for $n = 1, 2, 3, \dots$ then, $S(n) = S(n - 1) + X$. X is a random number with a given probability distribution under the following situations.

- ✦ X being positive implies an increase in the stock price
- ✦ X being negative implies a decrease in the stock price
- ✦ X is independent of the time. That means the changes in successive time periods are independent. [12]

Applying this model for our case study will be a challenging task as we don't have the historical stock prices to estimate the random number X .

3.8. The Geometric Brownian motion

The Geometric Brownian motion is today's most powerful method of modeling stock market movement that is widely used in financial option pricing. It was a model constructed based on the idea of the early 1900 famous study of the movement of the pollen grain called Brownian motion. Brownian motion, also called Brownian movement, is used to model any of various physical phenomena in which some quantity is constantly undergoing small, random fluctuations. It was named after the Scottish botanist Robert Brown, the first one to study such fluctuations (1827). [1]

Though it was originally used to describe random motion of particles, in 1900 Louis Bachelier's used this model to describe the random motion on stock prices in his doctoral dissertation. Bachelier proposed that the price of a stock moves like what is now called Brownian motion. This means that changes in the price over non-overlapping intervals of time are independent and Gaussian, with the variance of each price change proportional to the length of time involved. [8]

With the works of Albert Einstein and others, Brownian motion was considered as a continuous limit of a random walk. Norbert Wiener worked on this model and described the situation in terms of a probability measure over a space of continuous paths. As Wiener showed, it is legitimate to talk about a random real-valued continuous function W on $[0, \infty]$ such that

- $W(0) = 0$,
- for each $t > 0$, $W(t)$ is Gaussian with mean zero and variance t , and

- if the intervals $[t_1, t_2]$ and $[u_1, u_2]$ do not overlap, then the random variables $W(t_2) - W(t_1)$ and $W(u_2) - W(u_1)$ are independent. We now call such a random function a Wiener process. [4]

Even though the Brownian motion is used as a starting point for studying the movement of the stock market, it turns out that there is a negative price associated with it. This observation made by the British and American statisticians gave birth to the newly formulated Brownian motion called Geometric Brownian motion. The principal contribution was made in 1959 by the American astrophysicist M. F. Maury Osborne, who was the first to publish a detailed study of the hypothesis that $S(t)$ follows a geometric Brownian motion. [8] In short, Geometric Brownian motion is a variation of the Brownian motion where we consider the logarithm of the ratio of share price, rather than the price itself, so that the price will not be negative.

Based on the Wiener Process W , drift parameter μt , volatility parameter $\sqrt{t}\sigma$ and infinitely small positive real number dt , the stock price at time t can be modeled using the stochastic differential equation based on Brownian motion,

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \quad (3.8)$$

The solution to this differential equation can be formulated by simplifying equation (3.8), taking its integral and using Itô's lemma. [8] The process is indicated below:

Divide equation 3.8 by $S(t)$

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dW(t)$$

Take the integral of both sides.

$$\int_0^t \frac{dS(u)}{S(u)} = \int_0^t \mu du + \int_0^t \sigma dW(u)$$

$$= \mu t + \sigma[W(t) - W(0)]$$

Observe that $W(0) = 0$ from the Weiner process described above. Hence we have

$$\int_0^t \frac{dS(u)}{S(u)} = \mu t + \sigma W(t)$$

Using Itô's lemma to $F(S_t, t) = \log(S_t)$ yields the solution

$$S(t) = S(0)[e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W(t)}] \quad (3.9)$$

where $W(t)$ is a normal distribution with mean (drift parameter) μt and standard deviation $\sqrt{t} \sigma$.

To simplify our calculation and use the standard normal distribution, Equation (3.9) can be written as

$$S(t) = S(0)[e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma Z\sqrt{t}}] \quad (3.10)$$

Z follows the standard normal distribution with mean zero and standard deviation one.

In general the Geometric Brownian motion described above is built based on the following assumptions.

- ✦ The ratio of the price at time t (in the future) to the present price is independent of the past prices. [12]
- ✦ The ratio of prices has lognormal distribution with parameters drift μt and volatility $\sqrt{t}\sigma$. [12]

✚ Once μ and σ are determined, it is the present price- and not the past prices- that affects probabilities of the future prices. **[12]**

3.8.1. Using the Geometric Brownian Motion for Fellare Stock

In this section we will develop an Excel Spreadsheet Model by assuming that the Fellare Stock follows a Geometric Brownian Motion (GBM) and with the help of CB Analysis tool. The model building process is described below.

✦ Generating a Spreadsheet Model in Excel

The building of the Spreadsheet Model starts by identifying the parameters required for implementing the model. These parameters are listed in Table 3.10. The first column in the list shows the Cell names for the parameters identified. Each cell will have its own corresponding value generated by an assumption, or calculated by a formula or obtained from the case study as shown in Column two and three. The values for the Cell Names Current Price of the Stock and Strike Time were obtained directly from the case study. The Drift and Volatility Parameter were obtained from the calculation made on Page-8. The values associated with the cell name Z-Value will be generated by assuming a normal distribution with mean zero and standard deviation one. To verify the rightness of the model, a sample Z-Value was generated using Random Number Generation (RNG) data analysis tool of Excel 2010 (see Page-12 on how to Use RNG). The sample value generated was 0.3446. The estimated Expected Stock Price can be calculated by implementing Equation 3.10 as a formula in the cell. By using the sample Z-Value (0.3446) and the values for the other four parameters (indicated in Table 3.9), the Expected Stock Price was calculated to be \$43.91 which is the same as the value obtained from the spreadsheet model. This verifies the validity of our spreadsheet calculation. The next process will be defining the Assumption and Forecast Cells using the CB simulation analysis tools.

| Cell Name | Value | Value/ Formula/ Assumption required? |
|----------------------------|---------|--|
| Current Price of the Stock | \$42.00 | Value required: Case Study |
| Strike Time | 12 | Value required: Case Study |
| Drift Parameter | 0.00052 | Value required: Calculated on Page-8 |
| Volatility Parameter | 0.0416 | Value required: Calculated on Page-8 |
| Z-Value | 0.3446 | Assumption required: Normal Distribution |
| Expected Stock Price | \$43.91 | Formula required: Equation 3.10 |

Table 3.10: The Cell Names, Values, Formula and assumptions required to generate the spreadsheet model for Fellare Stock.

✦ **Defining the Assumption Cell**

The only assumption cell that we will have in this model will be the Z-Value cell. The Z-Value cell is defined as an assumption cell by using the Define Assumption tool of CB (See Page-13 on how to define Assumption Cells). The distribution for this assumption is normal probability distribution with mean zero and standard deviation of one.

✦ **Defining the Forecast Cell**

Once the assumption cell is defined, the next step will be defining the Forecast Cell. In our model, the only Forecast Cell is the Expected Stock Price Cell. And it can be defined using the Define Forecast tool of the CB (See Page-14 to the steps).

✦ **Running the Simulation and Analyzing the Result**

Now what is left is to run the simulation and analyze the result. To have a better simulation result, set the Number of trials to 100,000. This number indicates the number of random numbers that will be generated in our assumption cell (Z-Value) and also in our Forecast Cell (Expected

Stock Price). Before running the simulation I used the **Step** command to generate a single set of value so that I can verify the accuracy of the calculation. The single set of values generated for Z-Value and Expected Stock Price were 0.074807 and \$42.28, respectively. It appears that 0.074807 is a random number from a standard normal distribution and the value generated for the Expected Stock Price, \$42.28, is also a valid value (verified using Equation 3.10). After the verification, the simulation process is started by clicking on the **Start** command. Once the simulation is complete, the appropriate analysis tool from the CB control panel can be used for further graphical and numerical analysis of the result. Of the different analysis tools, I will present the analysis obtained from the Assumption, Forecast, Sensitivity, and Scatter Charts. I will also use the statistics and percentile tables generated for the analysis as part of the numerical analysis.

Assumption Chart

Of the charts generated, one of them is the Assumption chart. This chart is based on the 100,000 random numbers generated in the Z-Value cell. Figure 3.16 below shows the Assumption chart. Observe that the shape of this chart is bell-shaped, conforming that it is a standard normal distribution of mean zero and standard deviation of one. The statistics view of the chart indicates that the range of this normal distribution is [-4.23, 4.34].

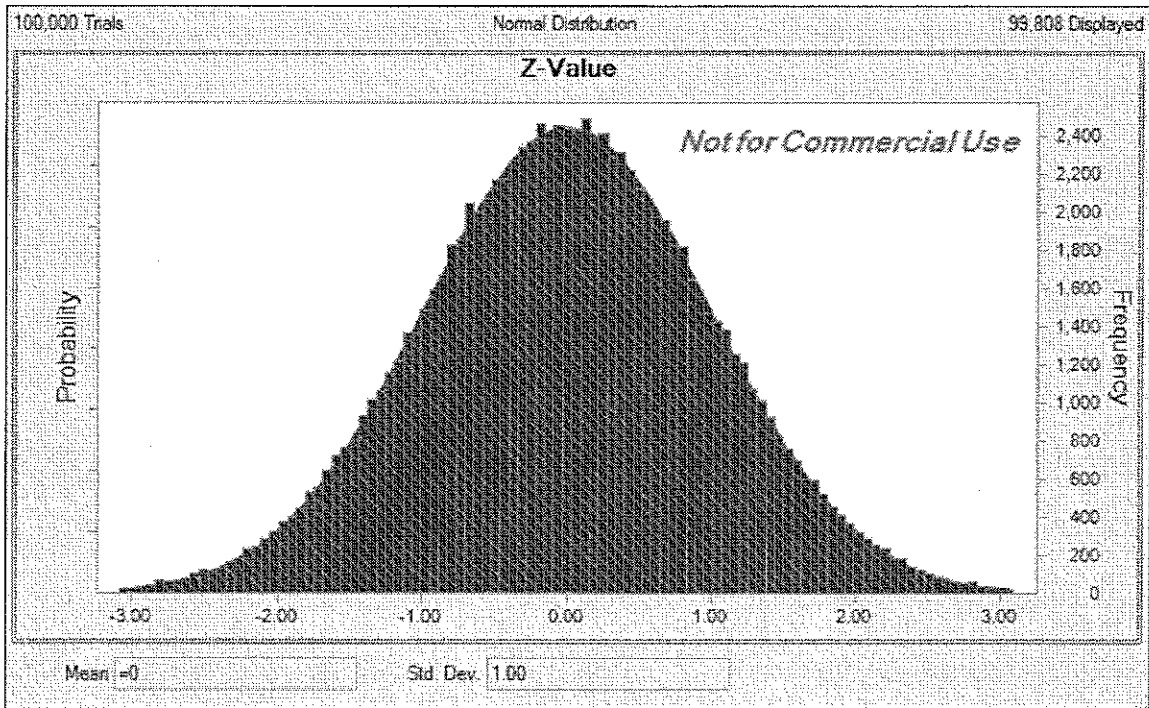


Figure 3.16: Normal distribution Assumption Chart with mean 0 and standard deviation of 1 for 100,000 randomly generated values.

Forecast Chart

The Forecast Chart is a chart generated for our Forecast Cell, Expected Stock Price. Figure 3.15 shows the Frequency View of the Forecast Chart for the Expected Stock Price. The chart indicates the Frequency, Probability, and Forecast Values with their certainty level and also the fitted curve for the graph. The Frequency (which is also the height of the curve) indicates how many simulations resulted over a certain range of the Forecast values for the Expected Stock Price. For example the tallest bar indicates that the Expected Stock Price was in the range in between \$42.17 and \$42.75 for around 4,500 trials. The probability to the left of the chart indicates the probability associated with the frequency. For example take the highest probability marked on the graph, 0.04. This number indicates that there is a 4% chance that 4000 trials of the simulation result will fall within a certain range. The certainty box indicated on the chart will

enable us to see the probability of achieving Forecast values with in a specific range. For example the certainty level that the Expected Stock Price is between \$41 and \$42 is 6.53%.

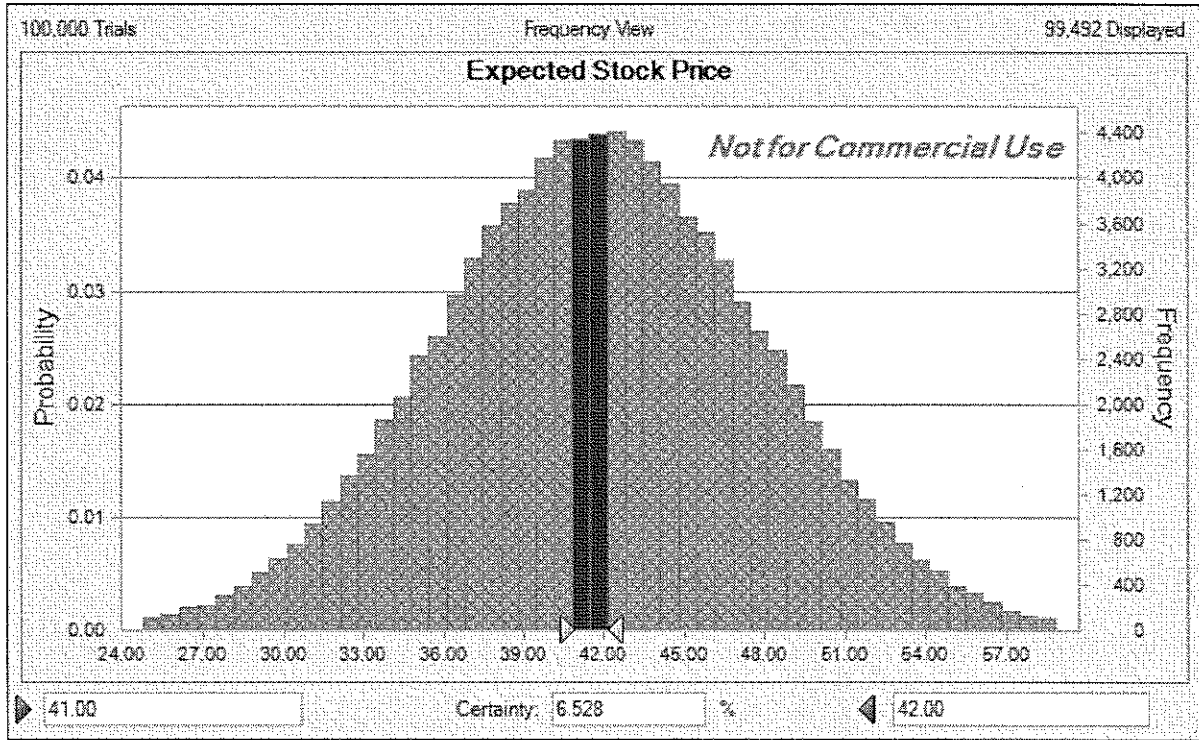


Figure 3.17: The Frequency View of the Forecast Chart for the Expected Stock Price indicating Probability, Frequency and Certainty level of the simulation run.

A full set of Descriptive statistics for the simulation result can be obtained from the Forecast Window (by selecting **View** then **Statistics**) as indicated in Table 3.11.

| Forecast: Expected Stock Price | |
|--------------------------------|-----------------|
| Statistic | Forecast values |
| Trials | 100,000 |
| Base Case | 41.83 |
| Mean | 41.81 |
| Median | 41.8 |
| Mode | --- |
| Standard Deviation | 6.08 |
| Variance | 36.93 |
| Skewness | -0.0093 |
| Kurtosis | 2.98 |
| Coeff. of Variability | 0.1454 |
| Minimum | 16.19 |
| Maximum | 68.07 |
| Mean Std. Error | 0.02 |

Table 3.11: Statistic Summary of the Forecast showing numerical values of the Forecast cell

With 100,000 numbers of simulated trials the mean of Expected Stock price was about \$41.81 with a standard deviation of \$6.19. The Median was \$41.8. The minimum and maximum forecast value for the Expected Stock Price was \$16.19 and \$68.07, respectively. The mean standard error, which shows the precision level of the simulation, was about 0.02.

Another useful analysis tool that we have is the Percentile View. The Percentile View can be obtained from the Forecast Window by selecting **View** and then **Percentile** as shown in Table 3.12. The percentile was calculated with a 10% increment. The 60th percentile indicates that there is a 60% chance of a forecast value being equal to or less than \$43.34.

| Forecast: Expected Stock Price | |
|--------------------------------|-----------------|
| Percentile | Forecast values |
| 0% | 16.19 |
| 10% | 34 |
| 20% | 36.72 |
| 30% | 38.63 |
| 40% | 40.28 |
| 50% | 41.8 |
| 60% | 43.34 |
| 70% | 44.99 |
| 80% | 46.94 |
| 90% | 49.62 |
| 100% | 68.07 |

Table 3.12 Percentile View of the Forecast Values calculated with a 10% increment

Sensitivity Chart

The Sensitivity Chart can be used to examine the level of the effect of the assumption on the Forecast values. Sensitivity Chart can be obtained from the Forecast Window by selecting the **Forecast Option** and then **Open Sensitivity Chart**. Figure 3.18 shows the Sensitivity Chart for the Expected Stock Price. From the chart observe that the only assumption we have is Z-Value, hence it accounts for 100% of the variance in the forecast values.

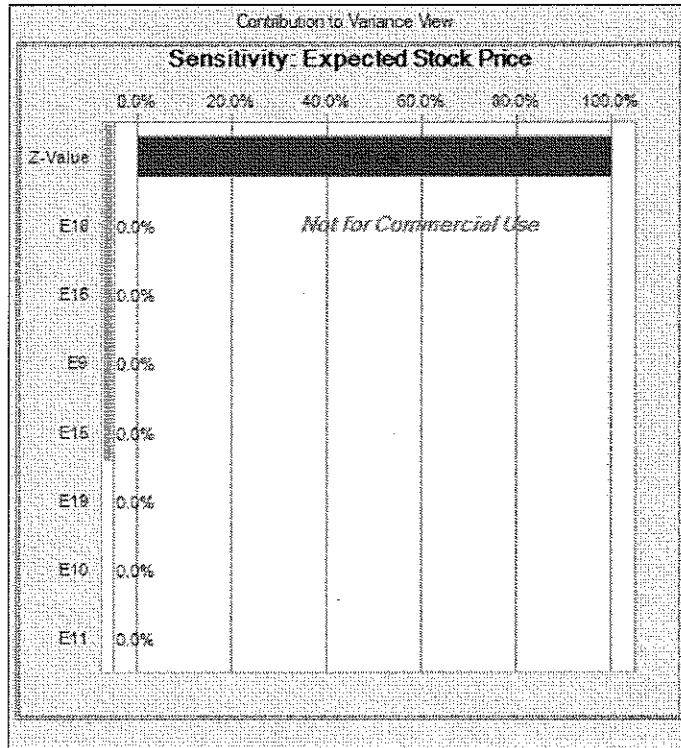


Figure 3.18: Sensitivity Chart for the Expected Stock Price.

Scatter Charts

Scatter Chart can be used to study the correlation between the Forecast Values and Assumption Values. Figure 3.19 shows the Scatter Chart for Expected Stock Price (Forecast Value) and Z-Value (Assumption Value). This chart was generated by clicking on the **CB Chart View** tab and followed by **Scatter Charts**. Then Click on New and select Expected Stock Price (set as Target) and Z-Value. The chart indicates that there is a positive correlation between the two values.

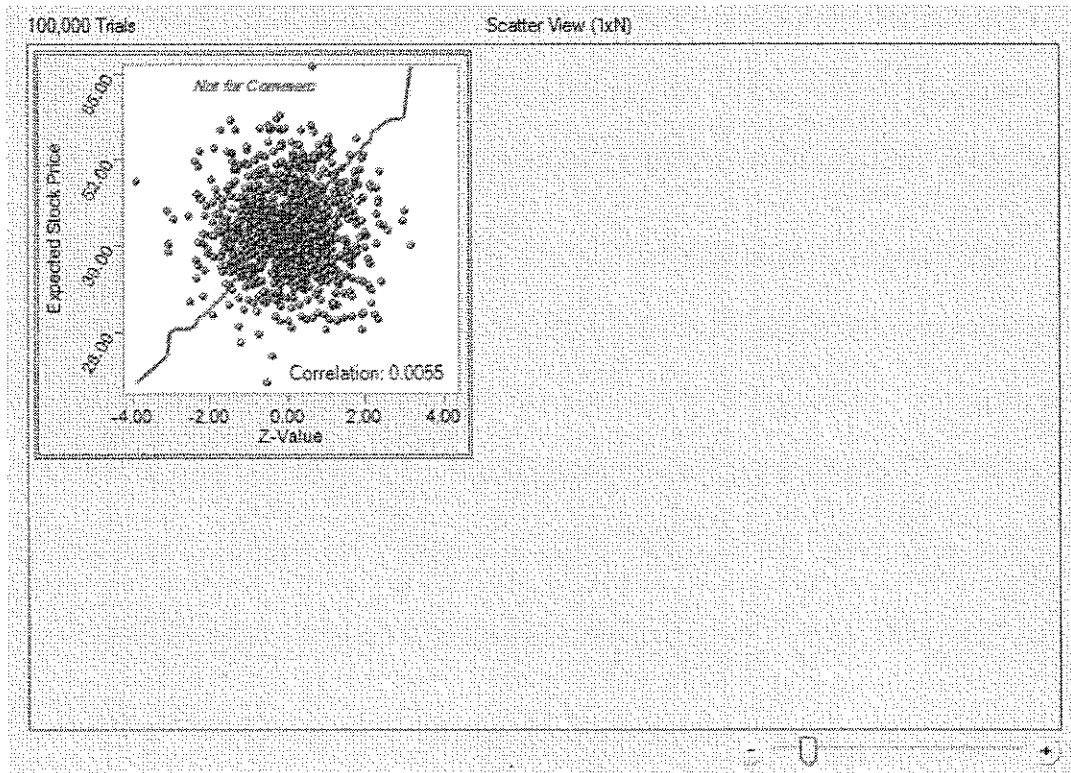


Figure 3.19: Scatter Chart indicating the correlation between Expected Stock Price and Z-Value

3.8.2. Using the Geometric Brownian motion for Intel Corporation

To see the validity of the application of GBM on stock prices, the model was used to forecast the stock price for a real stock on the market, Intel Corporation, and the forecasted price was compared with the actual current price on the market. The detail of the analysis is presented below.

To estimate the price of the stock, a spreadsheet model containing the six parameters was generated. The six parameters considered were Current Price of the Stock, Strike Time, Drift Parameter, Volatility Parameter, Z-Value and Expected Stock Price. These parameters were also used to define the cell names on the spreadsheet model as it was done in Section 3.7.1 (see page 11).

The value for the Current Price of the Stock was obtained from Yahoo Finance and the date considered was 02-22-2011. And on this date the closing price of INTC was recorded to be \$21.69. Strike Time will be twelve weeks from 02-22-2011 which will be on 05-16-2011.

The values associated with the Drift and Volatility parameters were obtained using the historical prices of INTC recorded in between 11-10-2008 and 11-8-2010 with a total number of 105 stock prices. Assuming that INTC prices will follow a lognormal distribution, the return value on the stock can be determined by taking the natural logarithm of the ratio of two consecutive stock prices, that is $\ln\left(\frac{S(n+1)}{S(n)}\right)$, where n indicates the number of stock price (ranges between 1 and 105). After finding the natural logarithm of the entire consecutive 105 stock price, the drift and volatility parameters were determined using the formulas $Drift = \frac{\sum_1^n\left(\frac{S(n+1)}{S(n)}\right)}{n}$ and $Volatility = \sqrt{\frac{1}{n-1}\sum_1^n(S(n) - Drift)^2}$, respectively. (See Page 23 and Table 3.3 for the details of the calculation). It turns the Drift and Volatility Parameters to be 0.005101 and 0.043813, respectively.

The value associated with Z- Value can be determined by generating random numbers that follows a standard Normal distribution. The steps in generating such a random number using Microsoft Excel are indicated in Section on page 13. By following the same steps a sample Z- Value of -0.4618 was generated.

Equation 3.10 was implemented in the Excel spreadsheet model to generate the values of the Expected Stock Price. This equation incorporates the values we have for the above five parameters also indicated in Table 3.13 below.

| Cell Name | Value | Value/ Formula/ Assumption required? |
|----------------------------|---------|--|
| Current Price of the Stock | \$21.69 | Value required: Yahoo Finance |
| Strike Time | 12 | Value required |
| Drift Parameter | 0.00510 | Value required: Calculated Using Historical Values |
| Volatility Parameter | 0.04381 | Value required: Calculated Using Historical Values |
| Z-Value | -0.4618 | Assumption required: Normal Distribution |
| Expected Stock Price | \$21.28 | Formula required: Equation 3.10 |

Table 3.13: The Cell names, the values, formula and assumptions used in generating the spreadsheet model for Intel Corporation

Once we have a working model of the Excel spreadsheet, we can use CB analysis tool to run the simulation for INTC stock. Before we run the simulation, we need to define the Assumption and Forecast Cell. Our Assumption Cell will be Z-Value Cell. It is defined by using the Define Assumption tool of CB and by selecting the normal distribution from the distribution gallery. (See page 13-14 on how to define assumption and forecast cells). The mean and standard deviation will be zero and one, respectively. Our Forecast Cell, Expected Stock Price, can also be defined using Define Assumption tool of the CB.

Before we run the simulation, it will be a good practice to check the accuracy of the model by generating a single set of values. Using the **Step** command available in CB, a value of -0.67747 and \$20.57 were generated for the Z-Value and Expected Stock Price, respectively. It appears that -0.67747 is a random number from a standard normal distribution table and \$20.57 is a valid value obtained using Equation 3.10. Hence our model is verified for accuracy and is ready for the simulation run.

Using 100,000 as the number of trials, the simulation can be started using the **Start** command of CB analysis tool. Once the simulation is complete, the result can be analyzed using the Assumption, Forecast, Sensitivity and Scatter Charts. And the Statistics and Percentile Tables can be used for further analysis. A discussion of the simulation result using these analysis tools is presented below.

Assumption Chart

The Assumption chart is based on the 100,000 random numbers generated in the Z-Value Cell. As expected, the shape of the graph generated is bell-shaped, confirming a standard normal distribution with mean zero and standard deviation of one as indicated in Figure 3.18. The range of random numbers generated, from the statistics view is [-3.87, 4.35].

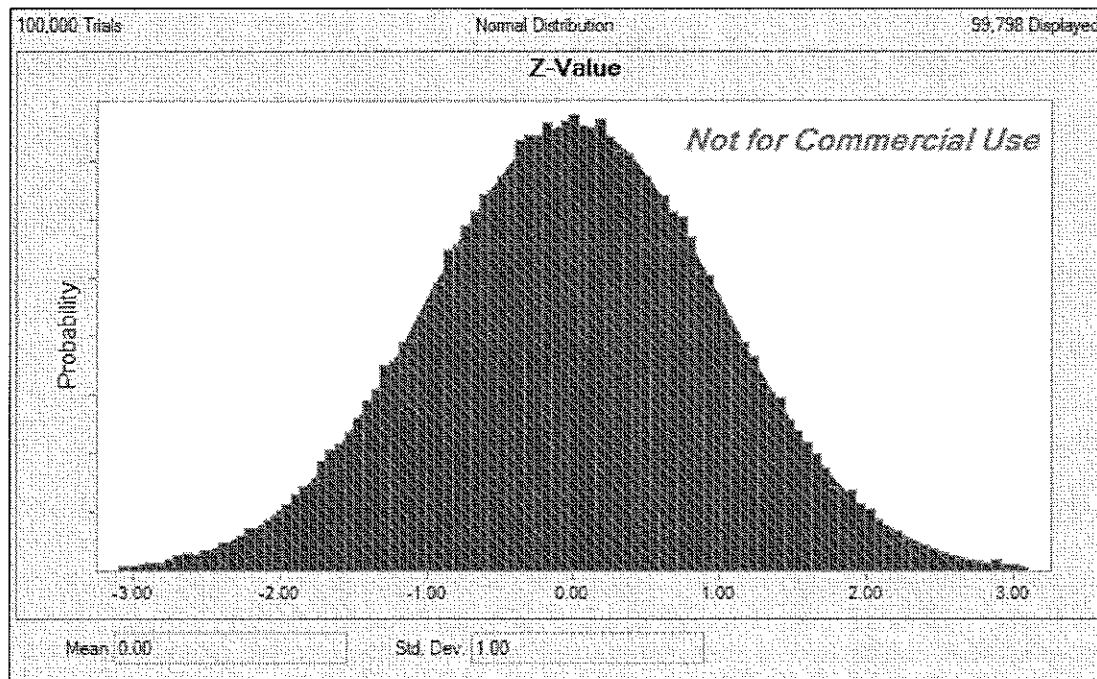


Figure 3.20: Normal Distribution Assumption Chart with mean 0 and standard deviation 1 for 100,000 randomly generated values for INTC stock evaluation

Forecast Chart

It is a chart generated based on 100,000 values of the Expected Stock Price. As indicated in Figure 3.19, I used the split view of the Frequency chart (Left Side), Statistics (Right Bottom) and Percentile (Right Top) tables to analyze the simulation result. The Frequency chart has three valuable components (Frequency, Probability and Certainty) for analyzing the result of our simulation. In this chart, the Frequency is represented by the height of the curve. It is the number of simulations resulted over a certain range for the 100,000 Forecast Values generated. For example, if we take the tallest bar in the chart, it indicates that around 4500 trials of the generated stock prices were priced between \$23.00 and \$23.36. The probability to the left of the chart indicates the probability that a certain number of trial occurs (calculated as: $\frac{\text{Number of Trials}}{\text{Total Number of Trials}}$). For example, the highest probability in the chart, 0.04, indicates that 4% of the trials of the simulation will fall within a certain range. The Certainty box can be used to determine the probability that the forecast value will fall within a certain range. For example, the certainty level that the Expected Stock Price will fall in between \$22 and \$23 is 11.78%.

The Statistics table indicates that for 100,000 numbers of trials the mean of the Expected Stock Price was about \$22.80 with a standard deviation of \$3.29. The median was found to be the same as the mean, \$22.80. Of the 100,000 values generated for the Expected Stock Price, the minimum price was \$9.42 and the maximum was \$35.99. The precision level of the simulation indicated as the mean standard error was about 0.01.

The percentile view explains the chance of a certain value of the Forecast to be less than or equal to the indicated values on the Percentile Table. For example, there is a 60% chance that the Expected Stock Price will have a price less than or equal to \$23.64.

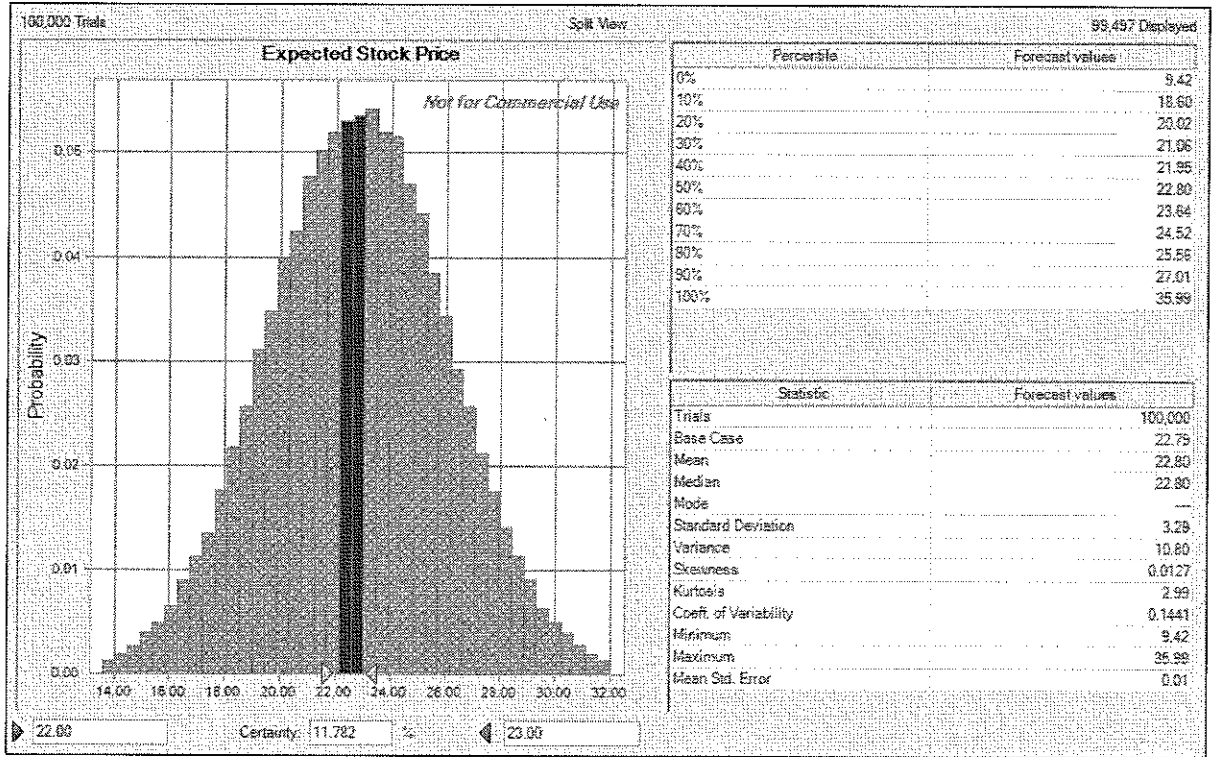


Figure 3.21: Split View of the Frequency View Chart (Left side), Percentile (right top) and Statistics (right bottom).

Sensitivity Chart

The sensitivity chart in Figure 3.22 indicates that our assumption (Z-Value) accounts for 100% of the variance in the Forecast Value (Expected Stock Price).

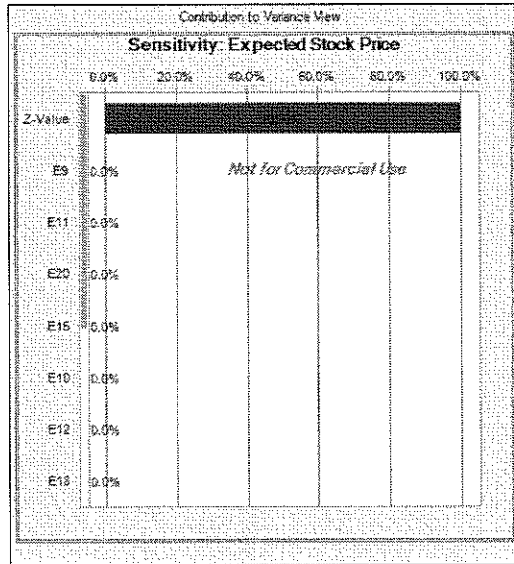


Figure 3.22: Sensitivity Chart on the Expected Stock Price for INTC stock

Scatter Chart

The Scatter Chart in Figure 3.23 indicates a positive correlation between the Z-Values and the Expected Stock-Price.

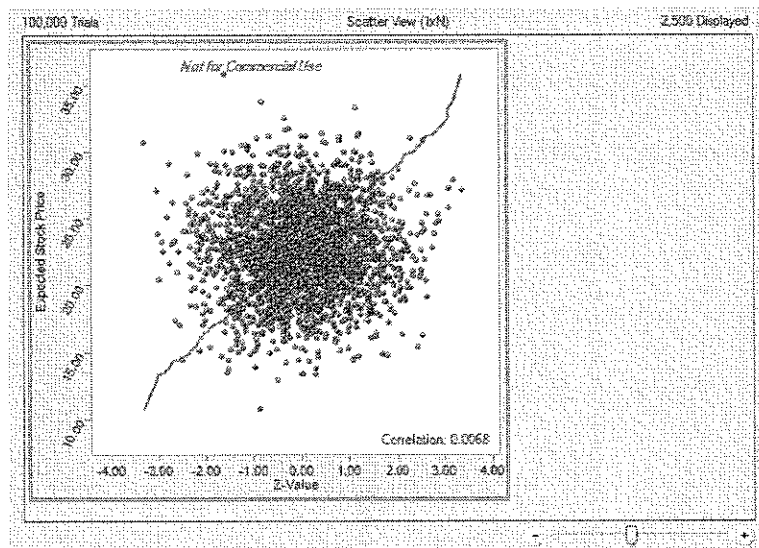


Figure 3.23: Scatter Chart for Expected Stock Price vs. the Z-Value for INTC stock.

Now let see the validity of the GBM by comparing the price we have from the simulation with the actual price on the Market. The Expected Stock Price on 5/16/2011 from our simulation result was \$22.80 (the mean of the 100,000 Forecast Values generated) and from Yahoo Finance the closing price on this specific date was \$23.22. This indicates that there is a very small (\$0.42) deviation from the forecasted price.

General Conclusion of Comparison of the Four Methods

Of the four methods of pricing the option presented, the literature reviews suggest the use of GBM to estimate stock price. The non –negativity of the prices it generates, the independence of the percentage changes, and the relative simplicity and good empirical fit all account for the popularity of GBM for simulating stock prices. [9] Just by taking a simple comparison of the values obtained (without going through a detailed statistical analysis) in our calculation, summarized in Table 3.14, it seems that the GBM model provides a better stock price prediction for the Intel Corporation. Even though we cannot generalize it as the best model for modeling stock prices based on Table 3.14, we can at least say under the given prices and conditions for the INTC stock the GBM provides a reasonable price. One of the set back of GBM model is the fact that it assumes a constant volatility parameter. But in real stock prices the volatility will vary indefinitely.

| Company | Lognormal | | Binomial | | GBM | |
|------------------|-------------------|-----------------|-------------------|-----------------|-------------------|-----------------|
| Name | Calculated | Observed | Calculated | Observed | Calculated | Observed |
| Fellare's | \$42.73 | n/a | \$42.75 | n/a | \$41.81 | n/a |
| Intel | \$23.34 | \$23.22 | \$21.69 | \$23.22 | \$22.80 | \$23.22 |

Table 3.14: Summary of the Stock price calculated using the Lognormal, Binomial and GBM

Models

4. Pricing the European Call Option

4.1. Introduction

A European call option will enable an investor to buy a share of stock on the strike date without the obligation to buy. [14] Having this right the buyer might be interested in knowing the expected payoff on a purchased option before exercising the option. If the stock price on the expiration date is higher than the exercise price, it is likely that the option will be exercised. So the decision in exercising the option will depend on the payoff of the purchased call option. We can calculate the payoff on a purchased call option as

$$\text{Max}\{0, (\text{the Stock Price on Expiration Date} - \text{Exercise Price})\} \quad (4.1)$$

The purchased payoff will be the maximum of 0 and the difference of the stock price and the exercise price. The option will be exercised if the payoff is greater than zero. For example on our case study, the strike price for Fellare Stock is \$43. The investor will exercise the option if the stock price on the strike date is higher than \$43. Any value less than \$43 will result in an option value of zero. This is described in the graph below.

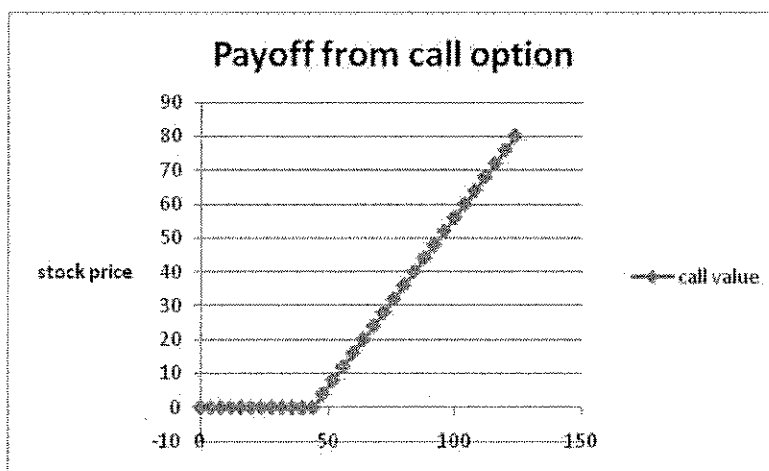


Figure 4.1: The Payoff from a call option of Fellare Stock with a strike price of \$43.

Having the payoff in mind, the next process will be pricing the call option. This process requires knowledge of statistical distributions as stock market movement are represented by probabilistic models. In Section 3.3, we discussed four different probabilistic models in studying the stock market movement. Also a spreadsheet model for each distribution was developed using the tools of Microsoft Excel and Crystal Ball. In this part of the project I will develop spreadsheet models for pricing European Call Option by considering four different approaches with their assumed probabilistic models. These approaches or techniques of pricing a European call options are

1. Option Pricing Via Arbitrage
2. Binomial Method of Option Pricing
3. Option Pricing Using the Black-Scholes Approximation Formulas
4. Monte-Carlo Simulation for Pricing an Option

4.2. Option Pricing Via Arbitrage

Before we consider the binomial method of pricing an option, let's see a means of pricing an option without setting specific assumptions on the stock market movement. In this discussion, we assume that the interest rate is compounded continuously and no dividend is provided on the stock. In addition we assume also that within one time period, the stock price will have either an up value of S_u or a down value of S_d . Consider the present value of the stock to be S_0 , the fair price of the option to be C and the exercise price of the stock to be k , where $S_d < k < S_u$.

At this point one might think of developing a situation in which we can minimize the risk of losing as the price of the stock varies between the two states. This situation can be explained by constructing a portfolio that will replicate the option. This replicating portfolio will

determine the number of shares of stock (x) to buy (sell), the number of options (y) to buy (sell) and the amount of money to borrow (B) so that regardless of the price of the stock we wouldn't lose.

The initial portfolio value (P_0) can be constructed based on the current price of the stock and the option as

$$P_0 = xS_0 + Cy \quad (4.2)$$

The future portfolio value can be constructed based on the outcome of the price of the stock.

If the price of the stock is up, the future portfolio (P_u) will be

$$P_u = xS_u + (S_u - K)y \quad (4.3)$$

If the price of the stock is below the strike price, the option will not be exercised and the future portfolio (P_d) will be

$$P_d = xS_d \quad (4.4)$$

To eliminate the uncertainty, we replicate the portfolio (Equate P_u and P_d) so that we wouldn't lose regardless of the stock price. Equating (4.3) and (4.4) and solving for y will yield

$$y = -\frac{S_u - S_d}{S_u - K} x \quad (4.5)$$

This replication of the portfolio, in light of Equation 4.5, will enable us to know how many shares of stock (x) to buy (sell) and also how many options (y) to buy or sell.

By considering an interest rate (r) compounded continuously over a short period of time (t), the initial portfolio will grow to

$$P_0 e^{rt} = (xS_0 + Cy)e^{rt} \quad (4.6)$$

To determine a fair value for the option, we will use equation (4.4), (4.5) and (4.6). First equate (4.4) and (4.6) to solve for C. Then in the resulting equation substitute the expression on the right side of equation (4.5) in place of y. This will yield

$$C = (S_0 - S_d e^{-rt}) \frac{S_u - k}{S_u - S_d} \quad (4.7)$$

Now we have a fair value C for the option. This technique of pricing the option is called Option pricing via Arbitrage. Arbitrage is the purchase of securities on one market for immediate resale on another in order to profit from a price discrepancy. [9] What if this theoretical price differs from the observed price? We can use the concept of arbitrage to compensate the misprice. Arbitraging the mispriced option will create a sure-win betting scheme to make a profit as follows. [12]

- ✦ If the observed value of the option is greater than the theoretical price -C (overpriced), buy stocks and sell options.
- ✦ If the observed value of the option is less than the theoretical price-C (underpriced), sell the stocks and short sell options.

Now let's see the application of this approach to our case study, the Felle stock. Table 4.1 summarizes the values associated with eight parameters obtained either directly from the case study or calculated in the previous section. The values for the parameters Current Price of the Stock, Strike Price, and Strike Time were directly obtained from the case study while the values

for Drift and Volatility Parameters, Interest Rate, Up-Value and Down-Value were obtained from calculations done on pages 8 and 31.

| Parameter | Value |
|-----------------------------------|----------|
| Current Price of the Stock $S(0)$ | \$42.00 |
| Strike Price (K) | \$43.00 |
| Drift Parameter (μ) | 0.00052 |
| Strike Time (in weeks(t) | 12.00000 |
| Volatility Parameter (σ) | 0.041603 |
| Interest rate(r : Weekly) | 0.148% |
| Up-Value (S_u) | \$43.85 |
| Down-Value (S_d) | \$40.35 |

Table 4.1: The eight parameters and their associated values required for pricing a European call option via Arbitrage

To determine the number of shares of stock(x) to buy (sell) and the number of options to buy (sell), we can use Equation 4.5 and the values we have in Table 4.1 as indicated below.

$$y = -\frac{S_u - S_d}{S_u - K}x$$

$$y = -\frac{43.85 - 40.35}{43.85 - 43}x$$

$$Y \approx -4X, \text{ that is, } 1Y \approx -4X$$

This indicates to buy 1 share of the Fellare stock and sell 4 shares of call options. The fair value, C , of the option is calculated using equation (4.7) and this value turns out to be a \$0.57.

4.3. Binomial Method of Option Pricing

Binomial method of option pricing simplifies the complex theory of option pricing as it is based on the binomial probabilistic distribution. The method assumes that the stock price will have only two states, that is, the price can go up or down within a specified period of time.

In this section a replicating portfolio on the call option payoffs will be constructed. The construction of such a portfolio requires finding a combination of stock and bonds so that the price of the option will be equal to the cost of replicating its payoffs. This portfolio will eliminate the uncertainty and can be used in determining the call premium. A brief discussion of the proposed portfolio is presented below.

Proposed Portfolio

We propose to construct a replicating portfolio by buying x shares of stock and borrowing $\$B$ at risk free-rate r .

Risk free-rate is the interest rate on a risk free investment. That means there is a very little (almost no) loss on the investment. A good example is the interest rate on a government bond.

Consider a stock market that has an up value of S_u and a down value of S_d . Let C_u and C_d represent the value of the option when the stock market goes up or down, respectively. Also assume a continuously compounded interest rate r over a period of time t with no dividend provided by the stock.

Let S_0 be the present value of the stock and consider u and d to be the factors by which the underlying price will go up or down respectively. Then the up and down values of the stock can be calculated as $S_u = uS_0$ and $S_d = dS_0$. The payoff on the proposed portfolio upon buying

x shares of stock and repaying a loan amount of Be^{rt} can now be calculated. When the price goes up, the payoff will be

$$xS_u - Be^{rt} \quad (4.8)$$

And when the price goes down, the payoff will be

$$xS_d - Be^{rt} \quad (4.9)$$

In a successful replicating portfolio, the price of the option must equal to the cost of replicating its payoff. That is, such a portfolio will satisfy

$$C_u = xS_u - Be^{rt} \quad (4.10)$$

$$C_d = xS_d - Be^{rt} \quad (4.10)$$

Using basic linear algebra and solving for x and B, we get

$$\begin{aligned} x &= \frac{C_u - C_d}{S_u - S_d} \\ &= \frac{C_u - C_d}{S_0(u-d)} \end{aligned} \quad (4.11)$$

$$\begin{aligned} B &= e^{-rt} \left[\frac{C_d S_u - C_u S_d}{S_u - S_d} \right] \\ &= e^{-rt} \left[\frac{uC_d - dC_u}{u-d} \right] \end{aligned} \quad (4.12)$$

So in this portfolio, the cost associated with the call option will be the amount needed to buy the stock and repay the loan amount. Hence the price of the option will be

$$C = xS_0 + B \quad (4.13)$$

Using equation (4.11) and (4.12) and rearranging the terms, equation (4.13) can be modified as shown below:

$$\begin{aligned} C &= xS_0 + B \\ &= \frac{C_u - C_d}{S_0(u-d)} (S_0) + e^{-rt} \left[\frac{uC_d - dC_u}{u-d} \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{C_u - C_d}{u - d} + e^{-rt} \left[\frac{uC_d - dC_u}{u - d} \right] \\
&= \left[\frac{C_u - de^{-rt}C_u}{u - d} \right] + \left[\frac{e^{-rt}uC_d - C_d}{u - d} \right] \\
&= C_u e^{-rt} \left[\frac{e^{rt} - d}{u - d} \right] + C_d e^{-rt} \left[\frac{u - e^{rt}}{u - d} \right] \tag{4.14}
\end{aligned}$$

Now observe that if we let $p = \frac{e^{rt} - d}{u - d}$, then $1 - p = \frac{u - e^{rt}}{u - d}$. This resembles the success and failure probability of a binomial model discussed in Section 3.4. Equation (4.14) can also be written in terms of p and $1 - p$ as

$$C = C_u e^{-rt} p + C_d e^{-rt} (1 - p) \tag{4.15}$$

According to Robert p is called the risk-neutral probability, the probability at which the stock and the option will be priced to provide the same riskless rate of return. [14]

Using the approximation of a binomial distribution to lognormal distribution, the up (u) and down (d) factors can be calculated using Equation (3.5) and (3.6).

4.3.1. Calculating the European Call Option for Fellare Stock

In Section 3.4.1, we were able to determine the Expected Stock Price for Fellare's Stock using the binomial approach. In this section we will attempt to price the European call option based on the Expected Stock Price determined earlier. The table below summarizes the calculation we did in section 3.4.1 by incorporating other information's provided on the case study. To start the option pricing process, we need to have a price or value for the eight parameters indicated in Table 4.2. From the parameters listed, the value for the Current Price of

the Stock and Strike Price were directly obtained from the case study. The values associated with the Interest rate, Drift and Volatility Parameters, Up and Down-Value Factor were obtained from the calculations made on Page 8 and Page 30. Note that the values we have for Strike Time, Drift and Volatility Parameters and Interest Rate are all weekly values.

| Parameters | Values |
|-----------------------------|---------------|
| Current Price of the Stock | \$42.00 |
| Strike Price | \$43.00 |
| Strike Time(For one period) | 1 |
| Drift Parameter | 0.000522 |
| Volatility Parameter | 0.041603 |
| Interest Rate | 0.148% |
| Up-Value Factor | 1.0440 |
| Down- Value Factor | 0.96067 |

Table 4.2: The eight parameters and their corresponding values required to price the call option for Fellare stock.

The binomial tree can be used to represent the possible values that an option may take at different binomial periods. By considering one week as one binomial period, the notion of pricing the call option week by week is presented below for the first two weeks. An Excel spreadsheet model for pricing the call option over the twelve-week period is also developed.

Construction of One –Period Binomial Tree

The one- period binomial tree on the call option price can be constructed by considering the payoff on the stock. The up and down values of the one period binomial (one-week) stock

prices were calculated in Section 3.4.1 and their respective values were \$43.85 and \$40.35 (see Table 3.6). The payoff on this period can be calculated using Equation 4.1 and this payoff will be the call option price of the stock on that specific period. The basic process is described below.

To use Equation 4.1 we need to know the Stock Price on Expiration Date and Strike Price of the Stock. The Stock Price on Expiration Date is either the Up or Down –Value of the stock, depending on the movement of the stock price and the Strike Price is \$43, obtained from Table 4.2. Now the call option price on the two possible values of the stock (that is the Up and Down – Values) can be calculated as follows.

- ✦ If Fellare’s stock price went up with in one period (week), the estimated Stock Price on Expiration will be \$43.85 and the payoff can be calculated as

$$\begin{aligned} & \text{Max}\{0, (\text{Stock Price on Expiration Date} - \text{Strike Price})\} \\ & = \text{Max}\{0, (\$43.85 - \$43)\} \\ & = \$0.85 \end{aligned}$$

Hence the value of the option when the stock price goes up is \$0.85. That is $C_u = \$0.85$.

- ✦ Were Fellare stock to go down within one week, the estimated Stock Price will be \$40.35 and the payoff will be:

$$\begin{aligned} & \text{Max}\{0, (\text{Stock Price on Expiration Date} - \text{Strike Price})\} \\ & = \text{Max}\{0, (\$40.35 - \$43)\} \end{aligned}$$

$$= \$0$$

In this case the value of the option when the stock price goes down is zero. (That is $C_d = \$0$)

Now the overall European call option price over a period of one week can be calculated using Equation 4.14. Observe that from the above calculations $C_u = \$0.85$ and $C_d = \$0$. The values for r, u, d and t can be directly obtained from Table 4.2. The calculation for the call option is indicated below

$$\begin{aligned} & C_u e^{-rt} \left[\frac{e^{rt} - d}{u - d} \right] + C_d e^{-rt} \left[\frac{u - e^{rt}}{u - d} \right] \\ &= \$0.85 e^{-(0.148\% * 1)} \left[\frac{e^{(0.148\% * 1)} - 0.96067}{1.0440 - 0.96067} \right] + 0 \left[e^{-rt} \left[\frac{u - e^{rt}}{u - d} \right] \right] \\ &= \$0.41 \end{aligned}$$

Hence the estimated one period European call option price on Fellare stock is \$0.41. The binomial tree for the option price is constructed using the Binomial Option Pricing Model code obtained from Wolfram Demonstration Project as shown below. [6] The looped structure on the figure shows that the Option Price is zero on that specific period.

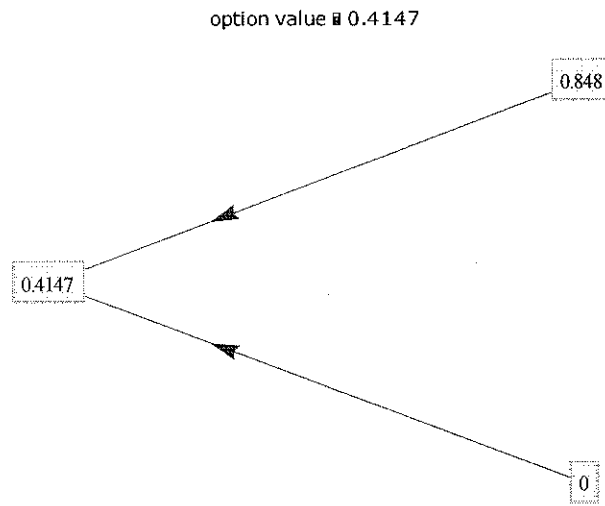


Figure 4.2: One-Period Binomial tree for Fellare's stock constructed using Mathematica. The source code was obtained from the Wolfram Demonstration Project title Binomial Option Pricing Model.

Construction of the Two- Period Binomial Tree

To construct the two-period binomial tree and calculate the European call option, we will work backwards starting with the four possible stock prices calculated at the end of the two-binomial period. The binomial tree for this period was indicated in Figure 3.9 and the values obtained from this tree are summarized in Table 4.3 below. In the table UP-UP, indicates that the price of the stock for two consecutive periods was UP. And UP-DOWN represents that the first period price was up and the second period price was down. Similar explanation also works for DOWN-UP and DOWN-DOWN.

| Stock Movement | Values |
|----------------|---------|
| UP-UP | \$45.78 |
| UP-DOWN | \$42.12 |
| DOWN-UP | \$42.12 |
| DOWN-DOWN | \$38.76 |

Table 4.3: The Possible stock prices after two-binomial periods obtained from the binomial tree diagram in Figure 3.9

Observe that the UP-DOWN and DOWN-UP prices are the same, as both of them follow similar trend. To price the two- period call option, I divided the process into three cases. The first case will be based on the stock prices obtained from the two-period binomial tree. The Second case will be based on the stock prices we have from the one-period binomial tree and also the Up and down option prices obtained from Case-1. Case-3 will be based on the up and down option prices recorded in Case-2. The process is described below.

Case 1: Week Two Call Option Price

To proceed with the calculation of the call option, consider the two- period stock prices of Fellare stock (indicated in Table 4.3 above). Since we are on the strike date, the call option for these prices will be the payoff on that specific period and it can be calculated using equation 4.1 as shown below. For the stock price UP-UP = \$45.78, the call option price (payoff) can be expressed as

$$\begin{aligned}
 \text{Call Option Price} &= \text{Max}\{0, (\text{Stock Price on Expiration Date} - \text{Strike Price})\} \\
 &= \text{Max}\{0, (\$45.78 - \$43)\} \\
 &= \$2.78
 \end{aligned}$$

For the stock price UP-DOWN = DOWN-UP = \$42.12, the price of the call option will be

$$\begin{aligned}
 \text{Call Option Price} &= \text{Max}\{0, (\text{Stock Price on Expiration Date} - \text{Strike Price})\} \\
 &= \text{Max}\{0, (\$42.12 - \$43)\} \\
 &= \$0
 \end{aligned}$$

For the stock price DOWN-DOWN = \$38.76, the price of the call option will be

$$\begin{aligned}
 \text{Call Option Price} &= \text{Max}\{0, (\text{Stock Price on Expiration Date} - \text{Strike Price})\} \\
 &= \text{Max}\{0, (\$38.76 - \$43)\} \\
 &= \$0
 \end{aligned}$$

A call option price of zero means that the option will not be exercised. Table 4.4 below summarizes the option prices calculated above.

| Stock Movement | European Call Option Price |
|----------------------------|----------------------------|
| UP –UP (45.78) | \$2.78 |
| UP-DOWN=DOWN-UP (\$42.12) | \$0 |
| DOWN-DOWN (\$38.76) | \$0 |

Table 4.4: The Expected European Call Option Price for the UP-UP, UP-DOWN, DOWN-UP and DOWN-DOWN stock movements

Case 2: Week One Call Option Price

To calculate the week one call option price, we will consider the one-period up and down stock prices and also the call option prices obtained in case-1 above. The one-period up and down prices can be obtained from the binomial tree constructed in Figure 3.9. The values are summarized in Table 4.5 and the option pricing process is presented below.

| Stock Movement | Value |
|----------------|---------|
| UP | \$43.85 |
| DOWN | \$40.35 |

Table 4.5: The Up and Down stock prices of one-binomial period obtained from the binomial tree diagram in Figure 3.8

Consider the UP price, \$43.85, being at this node the call option price associated with the up price (earlier denoted as UP-UP = \$45.78) was calculated to be \$2.78 (see Table 4.4) which can now be written as $C_u = \$2.78$. And the call option price associated with the down price (earlier denoted as UP-DOWN) was calculated to be \$0 which can now be written as $C_d = \$0$. Now the one-week -up-call option price can be calculated using Equation 4.14 as shown below.

$$\begin{aligned}
 & C_u e^{-rt} \left[\frac{e^{rt} - d}{u - d} \right] + C_d e^{-rt} \left[\frac{u - e^{rt}}{u - d} \right] \\
 &= \$2.78 e^{-(0.148\% \times 1)} \left[\frac{e^{(0.148\% \times 1)} - 0.96067}{1.0440 - 0.96067} \right] + 0 \left[e^{-rt} \left[\frac{u - e^{rt}}{u - d} \right] \right] \\
 &= \$1.36
 \end{aligned}$$

So the option price associated with the UP movement of the stock is \$1.36.

Now consider the DOWN price, \$40.35, again being at this node the call option associated with the up-price (earlier denoted as DOWN-UP) was calculated to be \$0. This value will be the Up option price for this node (that is $C_u = \$0$). Again being at this node the call option associated with the down-price (earlier denoted as DOWN-DOWN) was calculated to be \$0. This value will be the down call option price for this node (that is $C_d = \$0$). Hence the one week down-call option price will be \$0. Table 4.6 shows a summary of the one-week call option price.

| Stock Movement | European Call Option Price |
|----------------|----------------------------|
| UP | \$1.36 |
| Down | \$0 |

Table 4.6: The Expected European Call Option Price for the UP and DOWN stock movements

Case 3: Week Zero Call Option Price (Required Option Price)

Now what is left is to calculate the week zero call option price, *the required option price*. At this node the UP and DOWN stock prices are \$43.85 and \$40.35. From Table 4.6 above, the up and down call option prices were calculated to be $C_u = \$1.36$ and $C_d = 0$, respectively. The required European call option, C, can be calculated using Equation 4.14 as indicated below.

$$\begin{aligned}
 & C_u e^{-rt} \left[\frac{e^{rt} - d}{u - d} \right] + C_d e^{-rt} \left[\frac{u - e^{rt}}{u - d} \right] \\
 = & \$1.36 e^{-(0.148\% \cdot 1)} \left[\frac{e^{(0.148\% \cdot 1)} - 0.96067}{1.0440 - 0.96067} \right] + 0 \left[e^{-rt} \left[\frac{u - e^{rt}}{u - d} \right] \right] \\
 = & \$0.66
 \end{aligned}$$

Hence the expected two- period European call option price is \$0.66 and the binomial tree for option pricing constructed using Mathematica is indicated below. Again the source code was obtained from Wolfram Demonstrations Project Binomial Option Pricing Model. [6] The looped (ring) structure in the figure indicates that the call option price is zero for that specific period.

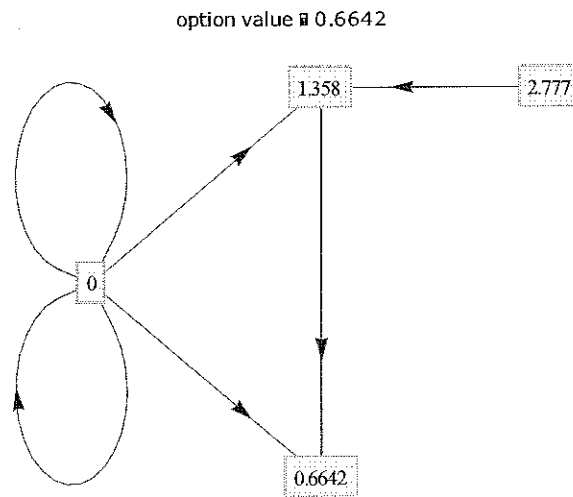


Figure 4.3: Two-Period Binomial tree for Fellare’s stock constructed using Mathematica. The source code was obtained from the Wolfram Demonstration Project title Binomial Option Pricing Model.

Following similarly procedures one can determine the Expected Value of European call option on Fellare’s stock after n-periods by constructing n-period binomial tree. Our interest here is to determine the European call option for Fellare’s stock after twelve-period (twelve weeks). Doing such a calculation for the twelve-periods and constructing a binomial tree will be a tedious and a very long process. As such I used Microsoft Excel to generate a twelve period binomial

option pricing spreadsheet model so that the twelve-period option price will be calculated with less effort. This spreadsheet model on its entirety will depend on the spreadsheet model developed in Section 3.6.1 for the binomial method of stock pricing (See Page 38-39). The process of developing the option pricing spreadsheet is described below.

The buildup process of the model can be accomplished by doing a backward calculation. That is the call option pricing process will be started from week twelve stock prices and goes all the way back to week one UP and DOWN prices. So we start with the last column stock price (week twelve prices) we have in Figure 3.13. Being at this node, the corresponding call option price for each stock price can be calculated using Equation 4.1. For example, for the first stock price, \$70.43(which is the UP-Value for that specific period), the call option price at this node is evaluated as

$$\begin{aligned} \text{Call Option Price} &= \text{Max}\{0, (\text{Stock Price on Expiration Date} - \text{Strike Price})\} \\ &= \text{Max}\{0, (\$70.43 - \$43)\} \\ &= \$27.43 \end{aligned}$$

This value will be the up- call option price for that specific period. (That is $C_u = \$27.43$). Again consider the next stock price we have from Figure 3.13, \$64.81. This price can be considered as the down- price for that specific period. And the call option price can be calculated as

$$\begin{aligned}
\text{Call Option Price} &= \text{Max}\{0, (\text{Stock Price on Expiration Date} - \text{Strike Price})\} \\
&= \text{Max}\{0, (\$64.81 - \$43)\} \\
&= \$21.81
\end{aligned}$$

This indicates that for this specific period $C_d = \$21.81$. Following similar procedures, we can calculate the call option price for the remaining stock prices we have in Figure 3.14. And the last column (Period -12) of the Excel spreadsheet model indicated in Figure 4.6 shows the calculated values of the remaining option prices for week twelve. When we do the backward calculation, the next step will be determining the call option prices for week 11. The option price for this period can be calculated based on the call option prices we have for week-12 and by using Equation 4.14. To find the first call option price we have for week -11, we will take the two option prices calculated above (that is, $C_u = \$27.43$ and $C_d = \$21.81$) and insert these values into Equation 4.14 as indicated below. Note that the values for r, u, d and t can be directly obtained from Table 4.2. We have

$$\begin{aligned}
&C_u e^{-rt} \left[\frac{e^{rt}-d}{u-d} \right] + C_d e^{-rt} \left[\frac{u-e^{rt}}{u-d} \right] \\
&= \$27.43 e^{-(0.148\%*1)} \left[\frac{e^{(0.148\%*1)}-0.96067}{1.0440-0.96067} \right] + 21.81 \left[e^{-rt} \left[\frac{u-e^{rt}}{u-d} \right] \right] \\
&= \$24.53
\end{aligned}$$

Hence our option price for this specific case will be \$24.53. Following similar procedures, we can calculate the remaining call option prices for week-11. This is indicated in column -11 of our Excel spreadsheet model. After doing this process recursively for the remaining twelve columns as shown in our spreadsheet model in Figure 4.6, the required

European call option price was determined. *This model estimates the price of European call option calculated over twelve binomial periods to be \$2.28.*

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
|---------|---------|---------|---------|---------|---------|----------|---------|----------|----------|----------|----------|----------|------|
| | | | | | | | | | | | | \$ 27.43 | |
| | | | | | | | | | | \$ 24.53 | | \$ 21.81 | |
| | | | | | | | | | \$ 19.08 | | \$ 19.14 | | |
| | | | | | | | | \$ 16.54 | | \$ 16.59 | | \$ 16.64 | |
| | | | | | | \$ 14.10 | | \$ 14.14 | | \$ 14.18 | | \$ 14.18 | |
| | | | | \$ 9.59 | | \$ 9.58 | | \$ 9.60 | | \$ 9.62 | | \$ 9.62 | |
| | | | \$ 7.59 | | \$ 7.52 | | \$ 7.47 | | \$ 7.47 | | \$ 7.49 | | |
| | | \$ 5.85 | | \$ 5.70 | | \$ 5.56 | | \$ 5.44 | | \$ 5.43 | | \$ 5.43 | |
| | \$ 4.38 | | \$ 4.19 | | \$ 3.98 | | \$ 3.75 | | \$ 3.52 | | \$ 3.46 | | |
| \$ 2.28 | \$ 3.20 | | \$ 2.98 | | \$ 2.75 | | \$ 2.47 | | \$ 2.14 | | \$ 1.69 | | |
| | \$ 1.40 | | \$ 1.20 | | \$ 0.97 | | \$ 0.71 | | \$ 0.40 | | \$ - | | |
| | | \$ 0.76 | | \$ 0.59 | | \$ 0.40 | | \$ 0.20 | | \$ - | | \$ - | |
| | | | \$ 0.35 | | \$ 0.22 | | \$ 0.10 | | \$ - | | \$ - | \$ - | |
| | | | | \$ 0.12 | | \$ 0.05 | | \$ - | | \$ - | \$ - | \$ - | |
| | | | | | \$ 0.02 | | \$ - | | \$ - | \$ - | \$ - | \$ - | |
| | | | | | | \$ - | | \$ - | \$ - | \$ - | \$ - | \$ - | |
| | | | | | | | \$ - | \$ - | \$ - | \$ - | \$ - | \$ - | |
| | | | | | | | | \$ - | \$ - | \$ - | \$ - | \$ - | |
| | | | | | | | | | \$ - | \$ - | \$ - | \$ - | |
| | | | | | | | | | | \$ - | \$ - | \$ - | |
| | | | | | | | | | | | \$ - | \$ - | |
| | | | | | | | | | | | | \$ - | |
| | | | | | | | | | | | | | \$ - |

Figure 4.6: Twelve-Period Binomial Spreadsheet Model indicating the Expected European call option price of \$2.28 for Fellare’s stock.

4.3.2. Calculating the call option for Intel Corporation

To see the appropriateness of the application of the binomial option pricing spreadsheet model constructed earlier, the model was implemented on a real stock market for Intel Corporation (INTC). The estimated price of the call option obtained from the model was crossed

checked with the value listed on Yahoo Finance. The implementation of the model is discussed below.

To use the spreadsheet model constructed earlier, we need to know the values associated with the eight parameters defined in column one of Table 4.2. Below is a discussion on how the values can be obtained.

For our analysis the INTC stock price on 5/27/11 was considered. On this date, which will be considered as Current Price of the Stock, the price on Yahoo Finance was recorded to be \$22.21. The Strike Time will be twelve weeks (twelve periods) from the above date. But for our calculation we will use the one period time (that is Strike Time of One). Again using yahoo finance, I recorded the actual market price of the call option (symbol INTC110820C00015000) twelve weeks from today (will be on 08/19/11) with a Strike Price of \$15 to be \$7.95.

The Drift and Volatility parameters were obtained by considering the historical prices of INTC dated from 11/10/2008 to 11/8/2010. This calculation was done in section 3.4.2 (See Page 22-23) and the calculated prices for Drift and Volatility parameters were 0.00510 and 0.043813, respectively.

The Interest Rate for our calculation will be the rate obtained from U.S. Department of Treasury. Over five years, this rate was determined to be 1.6% and the Weekly Interest Rate will be 0.0320% (see the calculation on Page 36). Using $r = 0.032\%$, $t = 1$ and $\sigma = 0.043813$ in Equation 3.5 and 3.6, the UP-and Down Value Factors were found to be 1.0451 and 0.9574, respectively. Table 4.7 summarizes the values associated with the eight parameters required for the Twelve-Period Binomial Option Pricing Spreadsheet Model generated earlier.

| Parameters | Values |
|-----------------------------|---------------|
| Current Price of the Stock | \$22.21 |
| Strike Price | \$15.00 |
| Strike Time(For one period) | 1 |
| Drift Parameter | 0.00510 |
| Volatility Parameter | 0.043813 |
| Interest Rate | 0.032% |
| Up-Value Factor | 1.0451 |
| Down- Value Factor | 0.9574 |

Table 4.7: The eight parameters and their corresponding values required to price the call option for INTC

Now to obtain the estimated European call option, the values we have in Table 4.7 were feed into the Twelve-Period Binomial Option Pricing Spreadsheet Model constructed earlier. The model estimated the price of the European call option for INTC with a Strike Price of \$15 and a strike date of 8\19\2011 to be \$7.27 as shown in Figure 4.7. Recorded on Yahoo Finance, the call option price with the same Strike date and Strike Price was \$7.95. It seems that there is a slight deviation (\$0.68) from what we calculated using the model developed. Further analysis on the deviation and error term will be made in Chapter 5.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---------|---------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| | | | | | | | | | | | | | \$ 22.71 |
| | | | | | | | | | | | | \$ 21.09 | \$ 19.55 |
| | | | | | | | | | | | \$ 19.54 | \$ 18.06 | \$ 16.65 |
| | | | | | | | | | | \$ 18.05 | \$ 16.64 | \$ 15.29 | \$ 14.00 |
| | | | | | | | | | \$ 16.63 | \$ 15.28 | \$ 13.99 | \$ 12.75 | \$ 11.56 |
| | | | | | \$ 12.73 | \$ 13.97 | \$ 15.27 | \$ 16.63 | \$ 17.98 | \$ 19.34 | \$ 20.71 | \$ 22.08 | \$ 23.45 |
| | | | | \$ 11.54 | \$ 12.73 | \$ 13.97 | \$ 15.27 | \$ 16.63 | \$ 17.98 | \$ 19.34 | \$ 20.71 | \$ 22.08 | \$ 23.45 |
| | | \$ 10.40 | \$ 11.54 | \$ 12.73 | \$ 13.97 | \$ 15.27 | \$ 16.63 | \$ 17.98 | \$ 19.34 | \$ 20.71 | \$ 22.08 | \$ 23.45 | \$ 24.82 |
| | \$ 9.31 | \$ 10.40 | \$ 11.54 | \$ 12.73 | \$ 13.97 | \$ 15.27 | \$ 16.63 | \$ 17.98 | \$ 19.34 | \$ 20.71 | \$ 22.08 | \$ 23.45 | \$ 24.82 |
| \$ 8.27 | \$ 9.31 | \$ 10.40 | \$ 11.54 | \$ 12.73 | \$ 13.97 | \$ 15.27 | \$ 16.63 | \$ 17.98 | \$ 19.34 | \$ 20.71 | \$ 22.08 | \$ 23.45 | \$ 24.82 |
| \$ 7.27 | \$ 8.27 | \$ 9.31 | \$ 10.40 | \$ 11.54 | \$ 12.73 | \$ 13.97 | \$ 15.27 | \$ 16.63 | \$ 17.98 | \$ 19.34 | \$ 20.71 | \$ 22.08 | \$ 23.45 |
| \$ 6.32 | \$ 7.27 | \$ 8.27 | \$ 9.31 | \$ 10.40 | \$ 11.54 | \$ 12.73 | \$ 13.97 | \$ 15.27 | \$ 16.63 | \$ 17.98 | \$ 19.34 | \$ 20.71 | \$ 22.08 |
| | \$ 5.42 | \$ 6.32 | \$ 7.27 | \$ 8.27 | \$ 9.31 | \$ 10.40 | \$ 11.54 | \$ 12.73 | \$ 13.97 | \$ 15.27 | \$ 16.63 | \$ 17.98 | \$ 19.34 |
| | | \$ 4.55 | \$ 5.42 | \$ 6.32 | \$ 7.27 | \$ 8.27 | \$ 9.31 | \$ 10.40 | \$ 11.54 | \$ 12.73 | \$ 13.97 | \$ 15.27 | \$ 16.63 |
| | | | \$ 3.73 | \$ 4.55 | \$ 5.42 | \$ 6.32 | \$ 7.27 | \$ 8.27 | \$ 9.31 | \$ 10.40 | \$ 11.54 | \$ 12.73 | \$ 13.97 |
| | | | | \$ 2.96 | \$ 3.73 | \$ 4.55 | \$ 5.42 | \$ 6.32 | \$ 7.27 | \$ 8.27 | \$ 9.31 | \$ 10.40 | \$ 11.54 |
| | | | | | \$ 2.23 | \$ 2.96 | \$ 3.73 | \$ 4.55 | \$ 5.42 | \$ 6.32 | \$ 7.27 | \$ 8.27 | \$ 9.31 |
| | | | | | | \$ 1.57 | \$ 2.23 | \$ 2.96 | \$ 3.73 | \$ 4.55 | \$ 5.42 | \$ 6.32 | \$ 7.27 |
| | | | | | | | \$ 0.99 | \$ 1.57 | \$ 2.23 | \$ 2.96 | \$ 3.73 | \$ 4.55 | \$ 5.42 |
| | | | | | | | | \$ 0.51 | \$ 0.99 | \$ 1.57 | \$ 2.23 | \$ 2.96 | \$ 3.73 |
| | | | | | | | | | \$ 0.17 | \$ 0.51 | \$ 0.99 | \$ 1.57 | \$ 2.23 |
| | | | | | | | | | | \$ 0.00 | \$ 0.17 | \$ 0.51 | \$ 0.99 |
| | | | | | | | | | | | \$ 0.00 | \$ 0.00 | \$ 0.17 |
| | | | | | | | | | | | | \$ 0.00 | \$ 0.00 |
| | | | | | | | | | | | | | \$ 0.00 |

Figure 4.7: Twelve-Period Binomial Spreadsheet Model indicating the Expected European call option price of \$7.27 for INTC stock

4.4. Option Pricing Using the Black-Scholes Approximation Formulas

Many of the option pricing methods and models we have today depend on the works of Fischer Black and Myron Scholes published on the Journal of Political Economy in 1973 at the University of Chicago. The Journal titled “The Pricing of Options and Corporate Liabilities”, presents a theoretical valuation formula for options, which is called “The Black-Scholes Formula”. The derivation of the formula was based on the assumption that: If options are correctly priced in the market, it should not be possible to make sure profits by creating portfolios of long and short positions in options and their underlying stocks. [5]

For the version of the formula presented below, the following assumptions are required.

- ✦ The stock price follows a lognormal distribution at the end of any finite interval.
- ✦ The nominal interest rate is compounded continuously and it is known and is a constant.
- ✦ The volatility of the stock return is a constant.
- ✦ It is possible to borrow at the risk free rate and to short sell at no cost and there is no transaction cost associated with buying or selling the stock or the option.
- ✦ The stock pays no dividend.

The Black-Scholes formula (BS) can be written as [14]

$$C(S(0), K, \sigma, r, t) = S(0)N(\varpi) - Ke^{-rt}N[\varpi - \sigma\sqrt{t}] \quad (4.16)$$

$$\text{where } \varpi = \frac{\log\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \quad (4.17)$$

The variables used are described below.

- ✦ $S(0)$: The current price of the stock.
- ✦ K : The strike price of the option.
- ✦ σ : The volatility of the stock.
- ✦ r : The continuously compounded risk-free interest rate
- ✦ t : The time to expiration
- ✦ $N(X)$: The standard normal distribution with mean 0 and variance of 1.

4.4.1. Using the Black-Scholes Formula For Fellare's Stock

To determine the price of the European call option for Fellare's stock using the Black-Scholes formula, a spreadsheet model was generated. The parameters required in the formula were directly obtained from Table 4.2 with slight change on the value of the Strike Time which will be 12 weeks in here. The continuously compounded risk-free interest rate (rate of return in a risk free investment) will have the same value as the Interest Rate used so far, as all of our models assume a risk free investment. The model calculates $(\varpi), (\varpi - \sigma\sqrt{t}), N(\varpi)$ and $N[\varpi - \sigma\sqrt{t}]$ separately as shown in Figure 4.8. $N(\varpi)$ and $N[\varpi - \sigma\sqrt{t}]$ were determined using the NORMSDIST () built in MS Excel.

| | |
|--|----------|
| Current Price of the Stock -S(0) | \$ 42.00 |
| Strike Price | \$ 43.00 |
| Volatility Parameter | 0.041603 |
| The Continuously Compounded risk-free interest rate -r | 0.148% |
| The Time to expiration - t | 12 |

| Parameters | Values |
|-------------------------------|-----------------|
| ω | 0.032018 |
| $\omega - \sigma \sqrt{t}$ | -0.1121 |
| | |
| $N(\omega)$ | 0.512771 |
| $N(\omega - \sigma \sqrt{t})$ | 0.455373 |
| | |
| Call Premium : C | 2.300062 |

Figure 4.8: Spreadsheet Model for Black Scholes Approximation Formula for Fellare's stock.

Our Spreadsheet model indicates that the price of the European call option with a Strike Price of \$43 and Strike Time of twelve weeks to be \$2.30.

4.4.2. Using the Black-Scholes Formula For Intel Corporation

To see the accuracy of the Black-Scholes formula in pricing a call option, I implemented the spreadsheet model developed earlier on Intel Corporation stock sticker (INTC). The INTC stock on 5/27/11 and with a Strike Time of twelve weeks and Strike Price of \$15, as recorded on Yahoo Finance, was used. [21] The parameters required as an input to the model were obtained from Table 4.7 with a slight change on the Strike Time (which will be 12) and the Interest Rate is the same as the continuously compounded risk-free interest rate. Figure 4.9 shows the implementation of the model and the call option price.

| | |
|--|----------|
| Current Price of the Stock -S(0) | \$ 22.21 |
| Strike Price | \$ 15.00 |
| The Volatility Parameter | 0.0438 |
| The Continuously Compounded risk-free interest rate -r | 0.0320% |
| The Time to expiration - t | 12 |

| Parameters | Values |
|--|-----------------|
| ω | 2.68803 |
| $\omega\text{-}\sigma\text{*sqrt}(t)$ | 2.536302 |
| | |
| $N(\omega)$ | 0.996406 |
| $N(\omega\text{-}\sigma\text{*sqrt}(t))$ | 0.994398 |
| | |
| Call Premium BS | 7.271452 |
| Actual Market C | 7.95 |
| Deviation | 0.678548 |

Figure 4.9: Spreadsheet Model for Black Scholes Approximation Formula for INTC stock.

The spreadsheet model for Black Scholes approximation formula calculated the expected European call option to have a price of \$7.27. There is a slight deviation of \$0.68 from the actual recorded price of the call option on Yahoo Finance on 08\19\11 (symbol INTC110820C00015000 and Priced \$7.95). [21] Further analysis on the error term is given in Chapter 6.

5. Monte Carlo Simulation for Pricing an Option

5.1. Introduction to Monte Carlo Simulation

Many of the mathematical models we have today rely on using random variables as an input parameters in forecasting real-life situation. And in many instances the outcomes associated with these random variables are unpredictable. One way of treating such type of unpredictability is by generating random values from a probabilistic model. Each of the random values generated can be used in forecasting the desired output in a repeated process called Simulation. Simulation allow us to study the complex mathematical equations associated with real-life systems in different disciplines including physics, chemistry, finance, and so on.

In this part of the project a well-known simulation model in finance, **Monte Carlo Simulation (MCS)** will be used in pricing a European call option. The use of Monte Carlo Simulation in pricing options was first published by Boyle (1977); but recently the literature in this area has grown rapidly. [9] The term Monte Carlo originated in a conversation between two Mathematicians employed by Los Alamos national Laboratory as a code word for their secret work on the atomic bomb. [9] Monte Carlo Simulation was named for Monte Carlo, Monaco, where the primary attractions are casinos containing games of chance. Games of chance such as roulette wheels, dice, and slot machines exhibit random behavior [4].

Earlier we used the Black-Scholes Formula to price a European call option which provided a closed form solution for the value of the call option. Now we will consider a different approach in pricing a European call option, the MCS method. The MCS method of pricing an option can demonstrate the efficiency of a formula as it incorporates the use of simulated random

values. The next two sections discuss the implementation of MCS on our case study (Fellare stock) and on Intel Corporations (INTC) stock.

5.2. Using Monte Carlo Simulation for the Fellare Stock.

Developing a spreadsheet simulation model using MCS requires series of steps to be completed. The detailed analysis of generating a Monte-Carlo Simulated spreadsheet is outlined below.

1. *Understanding the Problem and Identifying the Requirements to Build the Model*

The problem we have on hand is to develop a simulated model for pricing a European call option. In order to price the call option, we need to find out the values associated with each of the following parameters:

- ✓ Current Price of the Stock
- ✓ Strike Price of the Stock
- ✓ Strike Time
- ✓ Drift and volatility Parameters
- ✓ Weekly Interest Rate.

The values for Current Price of the Stock, Strike Price and Strike Time were directly obtained from our case study. Weekly Interest Rate, Drift and Volatility Parameters were obtained from the calculations done on Page-8. These values are summarized in Table 5.1 below.

| Parameters | Values |
|----------------------------|---------|
| Current Price of the Stock | \$42.00 |
| Exercise Price | \$43.00 |
| Drift Parameter | 0.00052 |
| Volatility Parameter | 0.0416 |
| Interest Rate | 0.148% |

Table 5.1: The five parameters and their values required to implement MCS model for Fellare's stock.

To start the process we need to assume that the stock price will follow a Geometric Brownian Motion and Equation (3.10) can be used to forecast the price of the stock. And in addition the values associated with the normally distributed random variables in equation (3.10) needs to be generated.

2. *Formulate the Simulation Model*

Formulation of the simulation model will require collecting and organizing the data that are needed. In addition to the data provided in the above table, we need to define an assumption cell and Forecast Cell.

✦ **Define Assumption Cell**

For the MCS model, we define one assumption cell named Z-Value and use the Define Assumption utility of CB (See Page-13 on how to Define an Assumption cell) to define the Z-Value Cell as the assumption cell on the spreadsheet. Z-Value will follow a normal distribution with mean zero and standard deviation one. We will generate as many random numbers as required for the Forecast Cell.

✦ Define Forecast Cell

For the MCS model, we define two forecast cells: one for the Expected Stock Price and the other for the European Call Option. The values associated with the Expected Stock Price can be obtained by implementing Equation 5.1 into this specific cell of the Excel spreadsheet. Equation 5.1 was proposed by slightly modifying Equation 3.10. It generates n independent replications of the stock price on a specific time by following GBM distribution. And using the CB analysis tool, Define Forecast, the Expected Stock Price Cell was defined as a Forecast Cell (See Page-14 on how to define a Forecast Cell).

$$S_{(t)}^{(i)} = S(0) \left[e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t} Z_{(t)}^{(i)}} \right], \quad (5.1)$$

for $i = 1, \dots, n$. And $Z_{(t)}^{(i)}$ is the standard normal random variable with mean 0 and standard deviation 1.

The Second Forecast Cell, European Call Option, uses the payoff formula to calculate the option price. The European call options can be calculated based upon the call option payoff as

$$C_t^i = \max\{S_{(t)}^{(i)} - K, 0\} \quad (5.2)$$

After implementing equation 5.2 into the European Call Option cell in the Excel Spreadsheet model, this cell was defined as the Forecast Cell using CB analysis tools (See page 14 on how to define a Forecast Cell). The next step will be verifying that the spreadsheet model is a working model and the output produced is a reflection of the formulas used. This verification was done by generating a single set of values using the **Step** command from the CB analysis tool. This command generated a sample Z-Value of 0.431061, Expected Stock Price of \$44.45 and a

European Call Option price of \$1.00. Calculations made by hand verified the rightness of the values generated. Hence our model is a working model and it is ready for a simulation run.

3. *Conduct the Simulation Runs and Analyze the Results*

To get a better simulation results, I set the Number of Trials to 100,000 and processed the simulation run. The result of the simulation run was analyzed using the following analysis tools.

✚ Assumption Chart

This chart was generated based on the values generated in our Assumption cell, Z-Value cell. As indicated in Figure 5.1, the chart generated resembles the shape of a normally distributed probability function as expected. The range of the Z-values generated was [-4.38, 4.26].

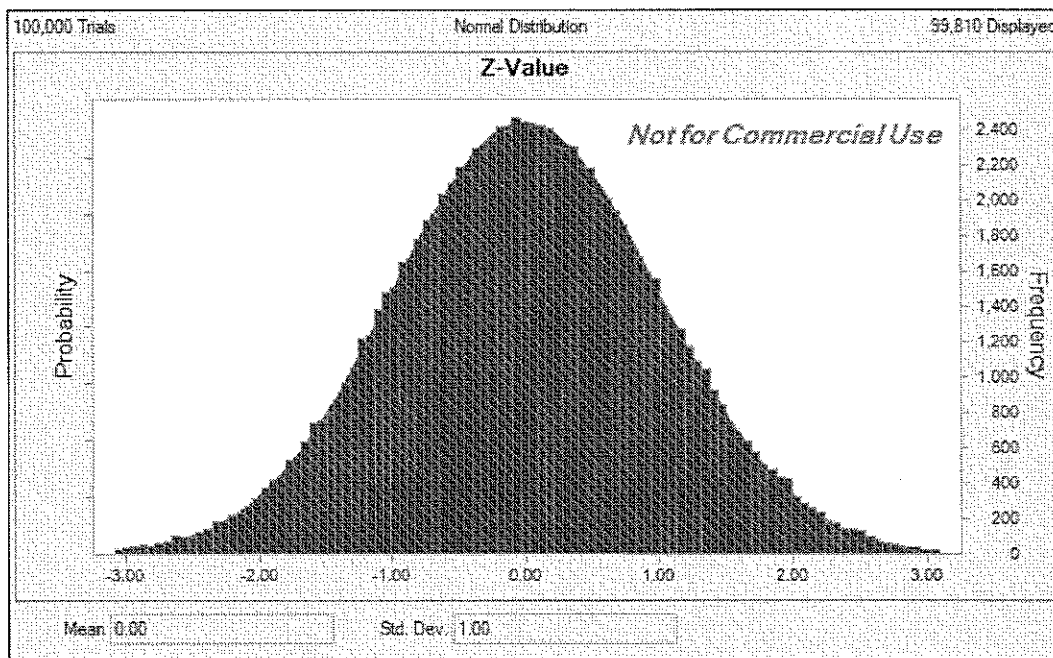


Figure 5.1: The Probability Assumption Chart for a Normal Distribution with mean zero and Standard Deviation one.

✦ Forecast Chart

As we have two Forecast Cells, there will be two different Forecast charts ready for analysis. But for the first chart, Expected Stock Price, I already presented a graphical and numerical analysis of the simulation result in Section 3.7.1(See Page 44-48 for details). So in this section I will only focus on the results obtained from our second forecast cell, European Call Option.

The Frequency view of this chart is presented in Figure 5.2 below. In here observe that the tallest bar (also called the Frequency) indicates that the European Call Option was zero for around 58,000 trials. The certainty box indicated on the figure explains that the probability of having a European Call Option between \$2 and\$3 is 5.47%.

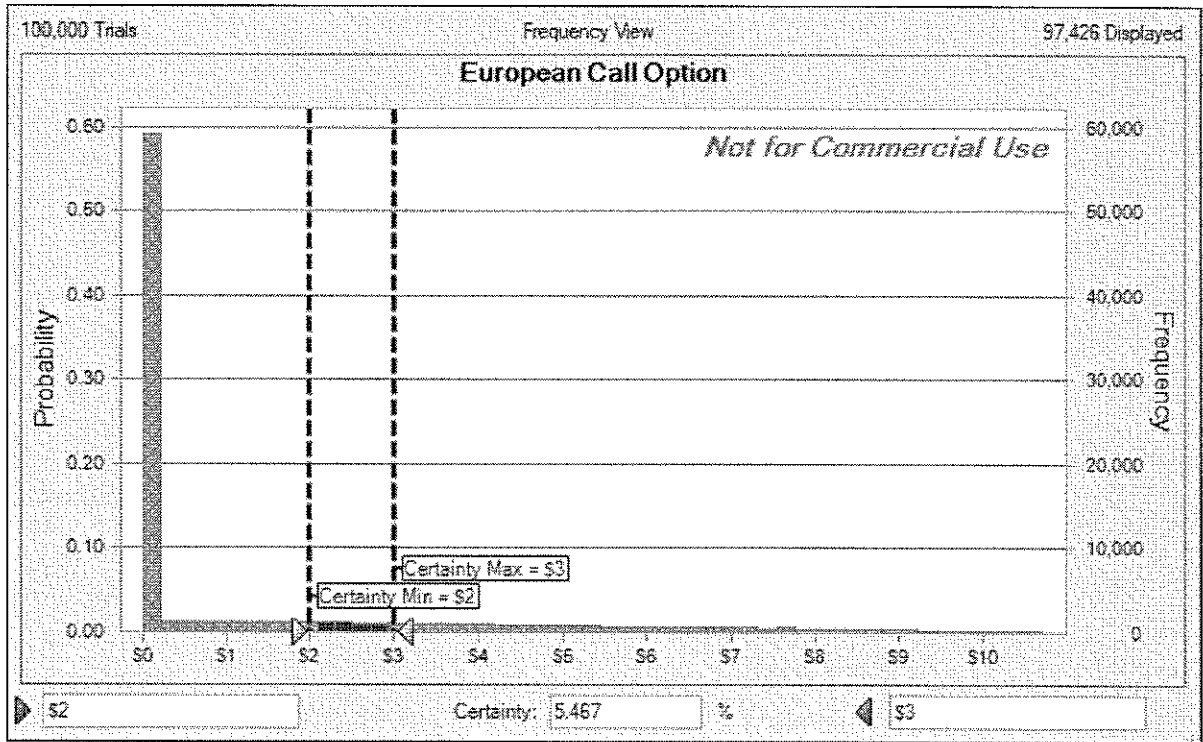


Figure 5.2: The Frequency View of the Forecast Chart for the European Call Option indicating Probability, Frequency and Certainty level of the simulation run.

The Statistics table, Table 5.2, presented below was used for numerical statistical analysis of the simulation result. *This table indicates that for the 100,000 number of trials, the mean European Call Option price was found to be \$2.00. The median was \$0 with a minimum price of \$0 and a maximum price of \$25. The mean standard error (shows the precision level of the simulation) was about 0.*

| Forecast: European Call Option | |
|--------------------------------|-----------------|
| Statistic | Forecast values |
| Trials | 100,000 |
| Base Case | \$0 |
| Mean | \$2 |
| Median | \$0 |
| Mode | \$0 |
| Standard Deviation | \$3 |
| Variance | \$10 |
| Skewness | 1.93 |
| Kurtosis | 6.54 |
| Coeff. of Variability | 1.67 |
| Minimum | \$0 |
| Maximum | \$25 |
| Mean Std. Error | \$0 |

Table 5.2: Statistics View of the Forecast for European Call Option price on Fellare's Stock

The Percentile View of the Forecast on the European Call Option , Table 5.3, indicates that there is a 60% chance of Forecast Value (the European Call Option Price) being equal to or less than \$0.

| Forecast: European Call Option | |
|--------------------------------|-----------------|
| Percentile | Forecast values |
| 0% | \$0 |
| 10% | \$0 |
| 20% | \$0 |
| 30% | \$0 |
| 40% | \$0 |
| 50% | \$0 |
| 60% | \$0 |
| 70% | \$2 |
| 80% | \$4 |
| 90% | \$7 |
| 100% | \$25 |

Table 5.3: Percentile View of the Forecast Values calculated with a 10% increment on European Call Option for Fellare's Stock.

✦ Sensitivity Chart and Scatter Charts

As the only assumption we have is the Z-Value, 100% of the variance in the Forecast value is dependent on the random values generated in this cell. This is also reflected in the Sensitivity Chart, Figure 5.3 below. The Scatter Chart (Figure 5.4) indicates that there is a very small correlation between the assumption (Z-Value) and Forecast (European Call Option). And there is a negative correlation between the two Forecast values (European Call Option and Expected Stock Price).

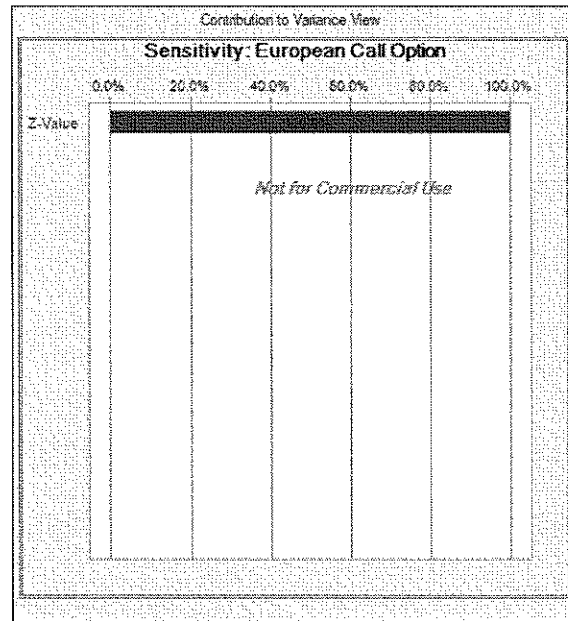


Figure 5.3: Sensitivity Chart for European Call Option Fellare's Stock.

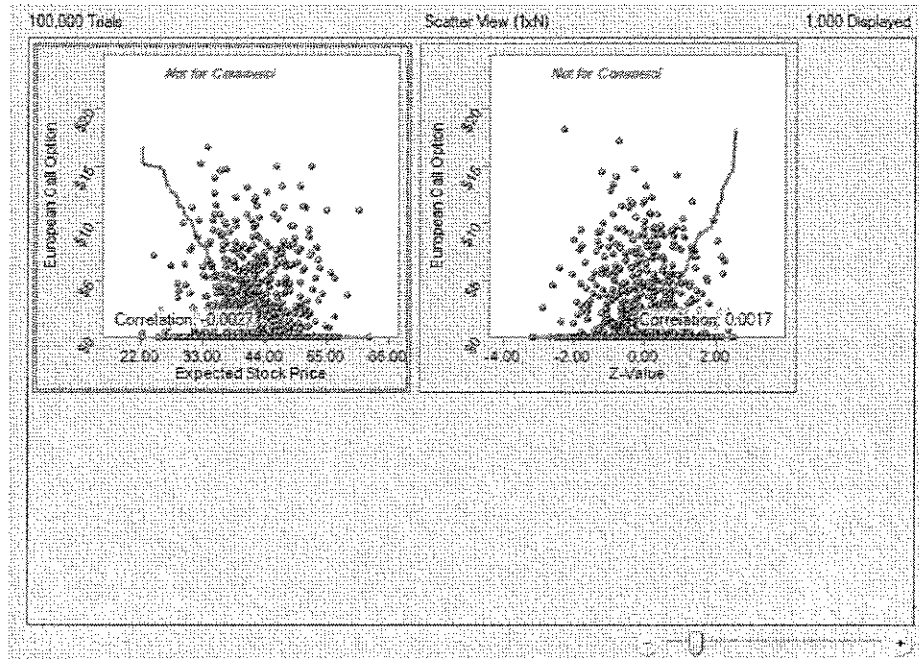


Figure 5.4: Scatter Chart indicating the correlation between European Call Option vs. Expected Stock Price and also European Call Option vs. Z-Value for Fellare's Stock

5.3. Using Monte Carlo Simulation for the Intel Corporation

To check the validity of MCS model generated in section 5.1.1, I used the model to estimate a European Call Option for a real stock on the market, Intel Corporation stock (INTC). For the analysis I used the INTC stock price on 5/27/11 with a closing stock price of \$22.21. Using Yahoo Finance, I recorded the Call Option price of INTC (Symbol: INTC110820C0015000) with a Strike Time of Twelve weeks and Strike Price of \$15 to be \$7.95. [21] The Drift and Volatility parameters were obtained using the weekly historical prices of INTC for two years (dated from 11/10/2008 to 11/8/2010). This calculation was done on Pages 22 and 23, and the calculated values were 0.000510 and 0.043813 for the Drift and Volatility Parameters respectively. The Interest Rate will be the rate from the U.S. Department of Treasury and it has a weekly value of 0.032% (see page 36 for detail). Table 5.4 summarizes the

values associated with the parameters discussed above. These values were used in MCS Spreadsheet Model constructed in Section 4.5.2 and the analysis on the result of the simulation model is presented below.

| Parameters | Values |
|----------------------------|----------|
| Current Price of the Stock | \$22.21 |
| Strike Price | \$15.00 |
| Drift Parameter | 0.0051 |
| Volatility Parameter | 0.043813 |
| Interest Rate | 0.032% |

Table 5.4: The Five Parameters and their values required to implement MCS model for INTC Stock

For the analysis, also in here the following three cells were used.

1. Assumption Cell: An assumption cell named Z-Value was defined. This Assumption Cell follows a normal distribution with mean zero and standard deviation one.
2. Forecast Cell- Expected Stock Price: A Forecast cell named Expected Stock Price was defined. The values for this cell were calculated using Equation 3.10 and by assuming that the stock price follows a Geometric Brownian Motion.
3. Forecast Cell- European Call Option: Another forecast cell named European Call Option was defined and the values for this cell were evaluated using Equation 5.2.

In this section of the study, our interest will be on the values generated for the Forecast Cell: European Call Option. Figure 5.5 shows a split view of the three analysis tools (Frequency View, Statistics and Percentile) used for analyzing the simulation result.

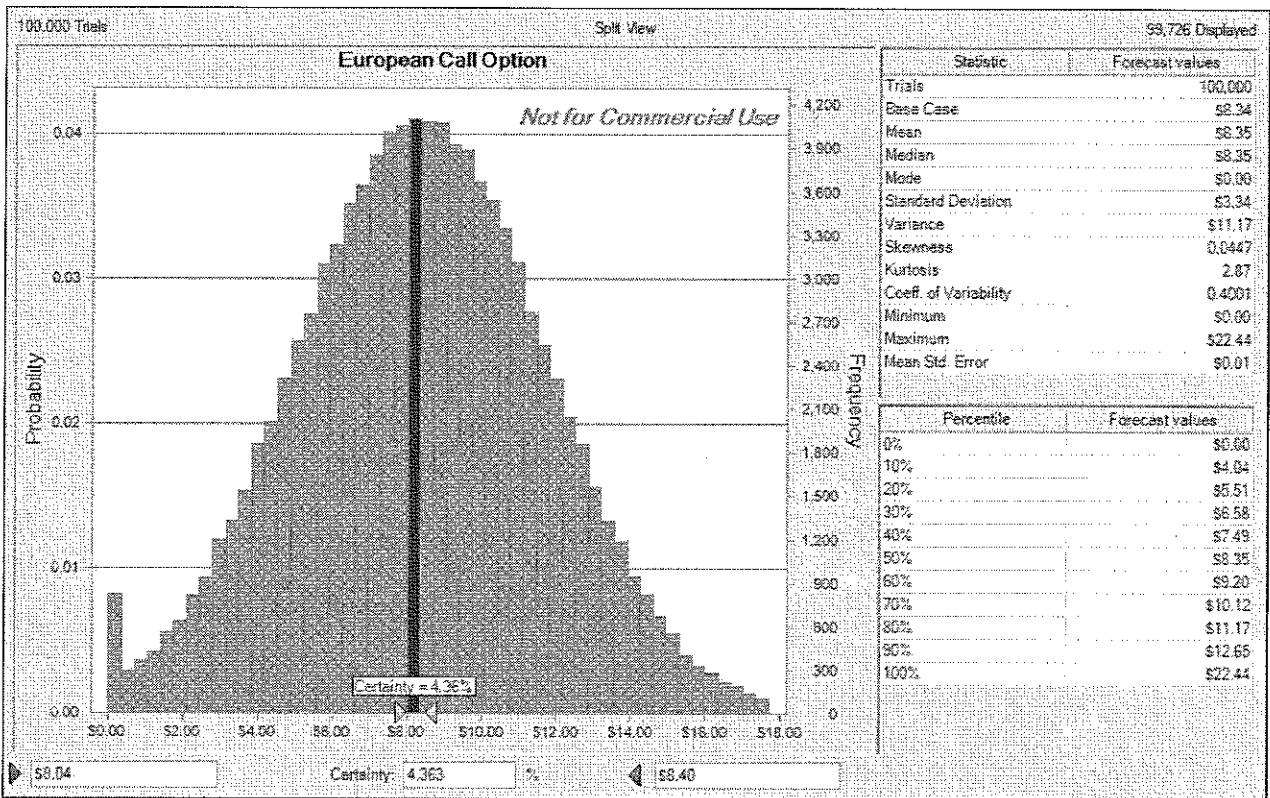


Figure 5.5: Split View of the Frequency View Chart (Left Side), Statistics View (Right Top) and Percentile View (Right bottom).

The Frequency view chart indicates that for almost the 4200 trials, the value of the European option was found to be in between \$8.04 and \$8.40. The certainty level for this range is 4.36%. The Statistics table indicates that for 100,000 trials, the mean European Call Option price was \$8.35. The median was also \$8.35. The minimum and maximum prices were observed to be \$0.00 and \$22.44 respectively. The error on the precision level of the simulation (that is the mean standard error) was about 0.01.

The Percentile view indicates that there is a 60% chance that for the European Call Option to be priced less than or equal to \$9.20. The Sensitivity Chart analysis (Figure 5.6) also indicated that 100% of the variation on the European Call Option Price was caused by the Z-Value (Assumption Cells) as Expected. The Scatter Chart analysis (Figure 5.7) indicates a negative correlation between European Call Option and Expected Stock Price.

The MCS Spreadsheet Model for INTC Stock estimated the expected price of the European Call Option to be \$8.35. The actual price of the Call Option Price recorded on Yahoo Finance, dated 08\19\11, with a strike price of \$15 and symbol INTC110820C00015000 was about \$7.95.

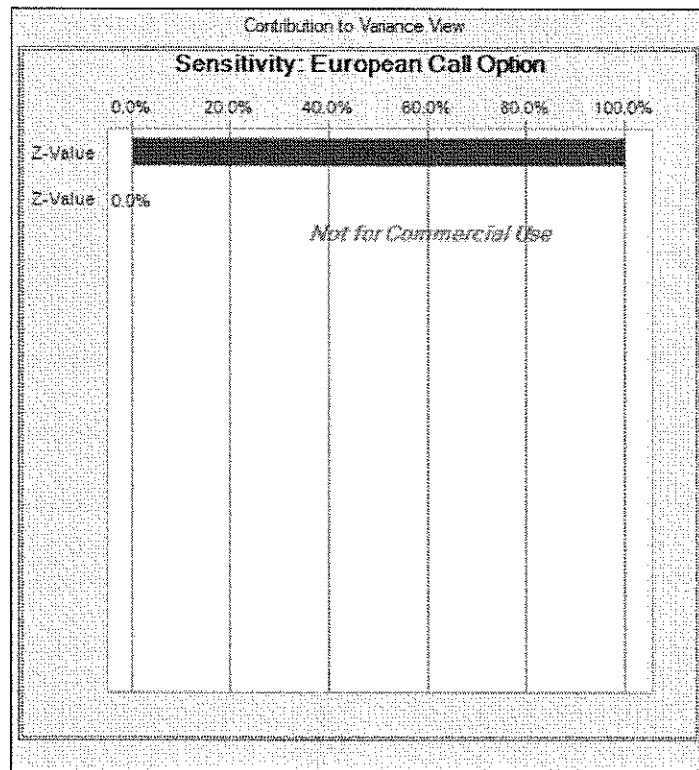


Figure 5.6: Sensitivity Chart for European Call Option on INTC Stock.

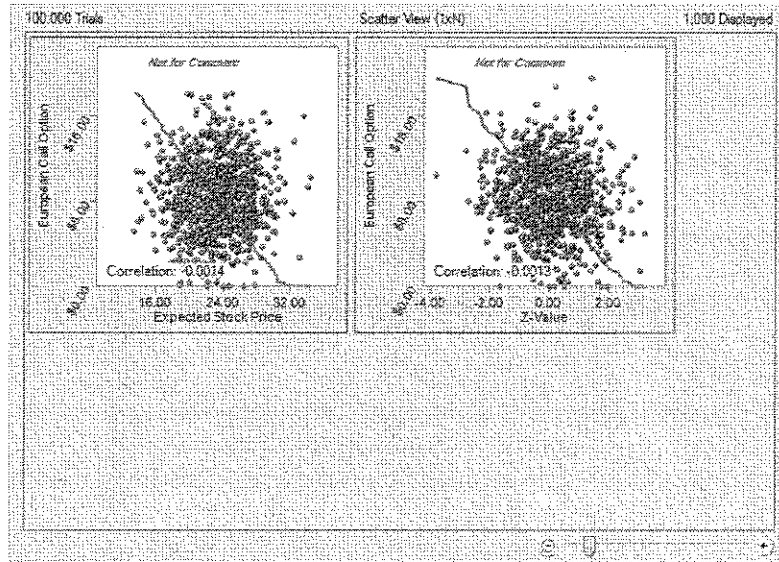


Figure 5.7: Scatter Chart indicating the Correlation between European Call Option vs. Expected Stock Price and also European Call Option vs. Z-Value for INTC Stock.

General Conclusion:

As the aim of this paper is to price the European call Option, it is imperative that we use a better model and provide the investor with a significant figure. With all the assumptions made and random values used in our models, it will be a challenging task to choose a better model. Table 5.5 summarizes the option values obtained from the three models considered. From this table it seems that it is inappropriate to prefer one model over the other as the deviations of the calculated price from the observed price (for INTC) under each model is very small. This suggests the need of further statistical analysis, which will be the topic of the next chapter. One observation that can be made from Table 5.5 is that the Binomial Model appears to give the same option price as the Black-Scholes Formula, which evidently supports the idea that a Binomial model can be approximated using the Black-Scholes model.

| Company | Binomial | | Black-Scholes | | MCS | |
|------------------|-------------------|-----------------|----------------------|-----------------|-------------------|-----------------|
| Name | Calculated | Observed | Calculated | Observed | Calculated | Observed |
| Fellare's | \$2.28 | n/a | \$2.30 | n/a | \$2.00 | n/a |
| Intel | \$7.27 | \$7.95 | \$7.27 | \$7.95 | \$8.35 | \$7.95 |

Table 5.5: Summary of the European Call Option price calculated using the Binomial, Black-Scholes and MCS Models.

6. Comparing the Black-Scholes and Monte Carlo Methods of Option Pricing

So far we considered four different approaches of pricing a European call option. The first two, the Arbitrage Approach and the Binomial Method of Option Pricing, are not widely used for real option pricing for a number of reasons. Even though the arbitrage method of option pricing is computationally simple and efficient, it is not widely used in option pricing as it requires the stock price to have only two values at the end of the period (up-or down). And the Binomial Method of Option Pricing is not used widely as it considers a multiple binomial process over a discrete period of time (say one day –one period) but in reality trading occurs in a continuously time manner. The other two, the Black-Scholes Formula and MCS, are widely used. The Binomial method can be approximated using the Black-Scholes formula by taking a very large number of periods. [14]

So in this part of the thesis, I will compare the efficiency and accuracy of the two models: MCS model and the Black Scholes formula. For comparison purpose, I considered the call option prices of forty-five different companies to compare the calculated call option price with the observed price. Table 5.1 shows the list of the companies under consideration. To make the study more inclusive, I classified the companies into three groups based up on the products and services they provide to customers as:

1. Financial Service Providers (FINANCIAL)
2. General Service Providers (SERVICES) and
3. Technology Oriented Companies (TECHNOLOGY)

| Financial | Services | Technology |
|--|-------------------------------------|------------------------------------|
| CITI Group, INC | Accenture plc | Apple Inc. |
| Capital One Financial Corp. | AT&T Inc. | Cisco Systems, Inc. |
| CIGNA Corporation | Barnes & Noble, Inc. Common Sto | Dell Inc. |
| Delphi Financial Group, Inc | Best Buy Co., Inc. Common Stock | General Electric Company Common |
| Hartford Financial Services Group INC | CVS Caremark Corporation | Hewlett-Packard Company Common |
| HSBC Holdings PLC (ADR) | Expedia, Inc. | Hitachi, Ltd. |
| Humana Inc. | FedEX Corporation Common Stock | Intel Corporation |
| JP Morgan Chase & C0.Common St | ITT Educational Services Inc. | International Business Machines |
| Mastercard Incorporated | J.C.Penney Company, Inc. | Microsoft Corporation |
| Morgan Stanley | John Wiley & Sons Inc. | Motorola Souldtions Inc |
| PNC Financial Services | Marriot International | Nokia Corporation |
| Prudential Financial, Inc. | McDonald's Corporation | Oracle Corporation |
| The Allstate Corporation | Starbucks Corporation | Siemens AG |
| Visa Inc | United Parcel Services, Inc. Com | Texas Instruments Incorporated |
| Waddell & Reed Financial, Inc | Wal-Mart Stores, Inc. Common St | Xerox Corporation |

Table 6.1: List of companies whose stock prices were considered under this study

For the analysis, the stock price on May-27-2011 was considered for each company. The actual option price twelve weeks from this date (which will be on August-19-2011) was recorded from Yahoo Finance. [20]

The parameters required to use the Black Scholes formula and MCS are summarized below:

✦ The Current Price of the Stock: This is the price of the stock under consideration on May-27-2011 and obtained from Yahoo Finance.[20]

✦ The Strike Price: To keep the uniformity of the study, I picked an option with a Strike Price over a certain range as shown below.

$$\text{Strike Price} = \{\text{Current Price of the Stock} * (1 + 0.02)\} \pm 2$$

✦ The Drift and Volatility Parameters: These parameters were calculated using the weekly historical prices of the companies dated between 11-10-2008 and 11-08-2010. (See Page 25 on how to calculate Drift and Volatility from historical data).

Note that the same date range was used for all the 45 stocks considered.

✦ Strike Time: For all the 45 stock prices considered, a strike period of twelve weeks was used.

✦ Weekly Interest Rate: The interest rate was obtained from the US Department of Treasury. And for all the stock prices under consideration, a weekly interest rate of 0.032% was used. (See Page 36)

The Excel Spreadsheets, Table 6.2 and Table 6.3, show the call option pricing process and the parameters required for the comparison process for the BS and MCS models respectively.

| Name of The Company | Stock Ticker | Symbol | Current Price of the Stock | Strike Price | Drift Parameter | Volatility Parameter | Calculated Model Option Price(MOP) | Observed Option Price(OOP) | Pricing Error (PE) | Squared PE PE | PE minus Mean of PE | Percentage Pricing Error PPE |
|---------------------------------------|-----------------|---------------------|----------------------------------|-----------------|--------------------|-------------------------|--|----------------------------------|--------------------------|---------------------|---------------------------|------------------------------------|
| Apple Inc. | AAPL | AAPL110820C00345000 | \$ 337.41 | \$345.00 | 0.01213 | 0.04761 | \$ 19.39 | \$ 20.02 | \$ 0.63 | \$ 0.40 | \$ 4.03 | 0.0324910 |
| Accenture plc | ACN | ACN110820C00057500 | \$ 56.91 | \$ 57.50 | 0.00470 | 0.03542 | \$ 2.61 | \$ 0.05 | \$ (2.56) | \$ 6.55 | \$ 0.84 | -0.9808429 |
| AT&T Inc. | T | T110820C00032000 | \$ 30.86 | \$ 32.00 | 0.00222 | 0.01941 | \$ 0.43 | \$ 0.01 | \$ (0.42) | \$ 0.18 | \$ 2.98 | -0.9767442 |
| Barnes & Noble, Inc. Common Sto | BKS | BKS110820C00020000 | \$ 19.56 | \$ 20.00 | 0.00027 | 0.07674 | \$ 1.91 | \$ 0.05 | \$ (1.86) | \$ 3.46 | \$ 1.54 | -0.9738220 |
| Best Buy Co., Inc. Common Stock | BBY | BBY110820C00032000 | \$ 31.44 | \$ 32.00 | 0.00712 | 0.06267 | \$ 2.52 | \$ 0.01 | \$ (2.51) | \$ 6.30 | \$ 0.89 | -0.9960317 |
| Capital One Financial Corp. | COF | COF110820C00055000 | \$ 53.97 | \$ 55.00 | 0.00142 | 0.12338 | \$ 8.80 | \$ 0.02 | \$ (8.78) | \$ 77.09 | \$ 5.38 | -0.9977273 |
| CIGNA Corporation | CI | CI110820C00050000 | \$ 49.47 | \$ 50.00 | 0.01009 | 0.08641 | \$ 5.74 | \$ 0.06 | \$ (5.68) | \$ 32.26 | \$ 2.28 | -0.9895470 |
| Cisco Systems, Inc. | CSCO | CSCO110820C00017000 | \$ 16.40 | \$ 17.00 | 0.00369 | 0.04347 | \$ 0.76 | \$ 0.01 | \$ (0.75) | \$ 0.56 | \$ 2.65 | -0.9868421 |
| CITI Group, Inc. | C | C110820C00042000 | \$ 40.96 | \$ 42.00 | -0.00731 | 0.18233 | \$ 9.83 | \$ 0.01 | \$ (9.82) | \$ 96.43 | \$ 6.42 | -0.9989827 |
| CVS Caremark Corporation | CVS | CVS110820C00040000 | \$ 38.67 | \$ 40.00 | 0.00081 | 0.04332 | \$ 1.80 | \$ 0.03 | \$ (1.77) | \$ 3.13 | \$ 1.63 | -0.9833333 |
| Dell Inc. | DELL | DELL110820C00016000 | \$ 15.79 | \$ 16.00 | 0.00261 | 0.05344 | \$ 1.10 | \$ 0.01 | \$ (1.09) | \$ 1.18 | \$ 2.31 | -0.9908842 |
| Delphi Financial Group, Inc | DFG | DFG110820C0003000 | \$ 28.72 | \$ 30.00 | 0.00847 | 0.09136 | \$ 3.13 | \$ 0.05 | \$ (3.08) | \$ 9.49 | \$ 0.32 | -0.9840256 |
| Expedia, Inc. | EXPE | EXPE110820C00028000 | \$ 27.69 | \$ 28.00 | 0.01235 | 0.07648 | \$ 2.83 | \$ 0.85 | \$ (1.98) | \$ 3.92 | \$ 1.42 | -0.6964666 |
| FedEX Corporation Common Stock | FDX | FDX110820C00095000 | \$ 91.76 | \$ 95.00 | 0.00339 | 0.05480 | \$ 5.70 | \$ 0.03 | \$ (5.67) | \$ 32.15 | \$ 2.27 | -0.9947368 |
| General Electric Company Common | GE | GE110820C00020000 | \$ 19.28 | \$ 20.00 | 0.00105 | 0.07061 | \$ 1.60 | \$ 0.01 | \$ (1.59) | \$ 2.53 | \$ 1.81 | -0.9977500 |
| Hartford Financial Services Group INC | HIG | HIG110820C00027000 | \$ 26.53 | \$ 27.00 | 0.00767 | 0.18567 | \$ 6.56 | \$ 0.02 | \$ (6.54) | \$ 42.77 | \$ 3.14 | -0.9969512 |
| Hewlett-Packard Company Common | HPQ | HPQ110820C00038000 | \$ 36.83 | \$ 38.00 | 0.00375 | 0.04293 | \$ 1.74 | \$ 0.01 | \$ (1.73) | \$ 2.99 | \$ 1.67 | -0.9942529 |
| Hitachi, Ltd. | HIT | HIT110820C00060000 | \$ 55.94 | \$ 60.00 | 0.00024 | 0.05075 | \$ 2.42 | \$ 0.70 | \$ (1.72) | \$ 2.96 | \$ 1.68 | -0.7107438 |
| HSBC Holdings PLC (ADR) | HBC | HBC110820C00052500 | \$ 51.71 | \$ 52.50 | 0.00129 | 0.06553 | \$ 4.41 | \$ 0.01 | \$ (4.40) | \$ 19.36 | \$ 1.00 | -0.9977324 |
| Humana Inc. | HUM | HUM110820C00082500 | \$ 79.15 | \$ 82.50 | 0.00640 | 0.08179 | \$ 7.65 | \$ 0.19 | \$ (7.46) | \$ 55.65 | \$ 4.06 | -0.951634 |
| Intel Corporation | INTC | INTC110820C00023000 | \$ 22.21 | \$ 23.00 | 0.00510 | 0.04381 | \$ 1.04 | \$ 0.01 | \$ (1.03) | \$ 1.06 | \$ 2.37 | -0.9903846 |
| International Business Machines | IBM | IBM110820C000170000 | \$ 167.50 | \$170.00 | 0.00615 | 0.03130 | \$ 6.39 | \$ 0.03 | \$ (6.36) | \$ 40.45 | \$ 2.96 | -0.9953052 |
| ITT Educational Services Inc. | ESI | ESI110820C00072500 | \$ 70.87 | \$ 72.50 | -0.00283 | 0.06687 | \$ 5.94 | \$ 0.08 | \$ (5.86) | \$ 34.34 | \$ 2.46 | -0.9865320 |
| J.C.Penney Company, Inc. | JCP | JCP110820C00037000 | \$ 35.79 | \$ 37.00 | -0.00097 | 0.05896 | \$ 2.45 | \$ 0.07 | \$ (2.38) | \$ 5.66 | \$ 1.02 | -0.9714286 |
| John Wiley & Sons Inc. | JW-A | JWA110820C00050000 | \$ 51.62 | \$ 50.00 | 0.00313 | 0.05264 | \$ 4.66 | \$ 0.10 | \$ (4.56) | \$ 20.79 | \$ 1.16 | -0.9785408 |
| JP Morgan Chase & Co. Common St | JPM | JPM110820C00043000 | \$ 42.53 | \$ 43.00 | 0.00176 | 0.09600 | \$ 5.49 | \$ 0.02 | \$ (5.47) | \$ 29.92 | \$ 2.07 | -0.9963570 |
| Marriott International | MAR | MAR110820C00039000 | \$ 37.86 | \$ 39.00 | 0.00904 | 0.06549 | \$ 2.99 | \$ 0.05 | \$ (2.94) | \$ 8.64 | \$ 0.46 | -0.9832776 |
| Mastercard Incorporated | MA | MA110820C00290000 | \$ 285.24 | \$290.00 | 0.00549 | 0.05553 | \$ 20.22 | \$ 20.00 | \$ (0.22) | \$ 0.05 | \$ 3.18 | -0.0108803 |
| McDonald's Corporation | MCD | MCD110820C000825000 | \$ 81.62 | \$ 82.50 | 0.00398 | 0.02426 | \$ 2.47 | \$ 5.55 | \$ 3.08 | \$ 9.49 | \$ 6.48 | 1.2469636 |
| Microsoft Corporation | MSFT | MSFT110820C00025000 | \$ 24.76 | \$ 25.00 | 0.00322 | 0.04040 | \$ 1.31 | \$ 0.02 | \$ (1.29) | \$ 1.66 | \$ 2.11 | -0.9847328 |
| Morgan Stanley | MS | MS110820C00025000 | \$ 24.02 | \$ 25.00 | 0.00812 | 0.09089 | \$ 2.64 | \$ 0.01 | \$ (2.63) | \$ 6.92 | \$ 0.77 | -0.9962121 |
| Motorola Solutions Inc | MSI | MSI110820C00049000 | \$ 47.58 | \$ 49.00 | 0.006768 | 0.069581 | \$ 4.03 | \$ 0.45 | \$ (3.58) | \$ 12.84 | \$ 0.18 | -0.8884205 |
| Nokia Corporation | NOK | NOK110820C00009000 | \$ 8.20 | \$ 9.00 | -0.00085 | 0.05810 | \$ 0.37 | \$ 0.02 | \$ (0.35) | \$ 0.12 | \$ 3.05 | -0.9459459 |
| Oracle Corporation | ORCL | ORCL110820C00034000 | \$ 33.64 | \$ 34.00 | 0.005351 | 0.038193 | \$ 1.67 | \$ 0.04 | \$ (1.63) | \$ 2.66 | \$ 1.77 | -0.9760479 |
| PNC Financial Services | PNC | PNC110820C00062500 | \$ 62.16 | \$ 62.50 | -0.00047 | 0.10651 | \$ 9.06 | \$ 0.05 | \$ (9.01) | \$ 81.18 | \$ 5.61 | -0.9944812 |
| Prudential Financial, Inc. | PRU | PRU110820C00065000 | \$ 63.17 | \$ 65.00 | 0.00796 | 0.12183 | \$ 9.92 | \$ 0.03 | \$ (9.89) | \$ 97.81 | \$ 6.49 | -0.9969758 |
| Siemens AG | SI | SI110820C00130000 | \$ 128.19 | \$130.00 | 0.00826 | 0.05575 | \$ 9.27 | \$ 0.06 | \$ (9.21) | \$ 84.82 | \$ 5.81 | -0.9935275 |
| Starbucks Corporation | SBUX | SBUX110820C00037000 | \$ 36.46 | \$ 37.00 | 0.01229 | 0.05230 | \$ 2.45 | \$ 0.03 | \$ (2.42) | \$ 5.86 | \$ 0.98 | -0.9877551 |
| Texas Instruments Incorporated | TXN | TXN110820C00035000 | \$ 34.57 | \$ 35.00 | 0.00681 | 0.04416 | \$ 1.97 | \$ 0.02 | \$ (1.95) | \$ 3.80 | \$ 1.45 | -0.9898477 |
| The Allstate Corporation | ALL | ALL110820C00032000 | \$ 31.24 | \$ 32.00 | 0.00200 | 0.06246 | \$ 2.41 | \$ 0.01 | \$ (2.40) | \$ 5.76 | \$ 1.00 | -0.9958506 |
| United Parcel Services, Inc. Com | UPS | UPS110820C00075000 | \$ 72.96 | \$ 75.00 | 0.00314 | 0.04256 | \$ 3.52 | \$ 0.01 | \$ (3.51) | \$ 12.32 | \$ 0.11 | -0.9971591 |
| Visa Inc | V | V110820C00080000 | \$ 79.80 | \$ 80.00 | 0.00369 | 0.04907 | \$ 5.46 | \$ 0.95 | \$ (4.51) | \$ 20.34 | \$ 1.11 | -0.8260073 |
| Waddell & Reed Financial, Inc | WDR | WDR110820C00040000 | \$ 38.11 | \$ 40.00 | 0.01079 | 0.07665 | \$ 3.31 | \$ 0.25 | \$ (3.06) | \$ 9.34 | \$ 0.34 | -0.9243799 |
| Wal-Mart Stores, Inc. Common St | WMT | WMT110820C00055000 | \$ 54.70 | \$ 55.00 | 0.00081 | 0.02730 | \$ 2.02 | \$ 0.02 | \$ (2.00) | \$ 4.00 | \$ 1.40 | -0.9900990 |
| Xerox Corporation | XRX | XRX110820C00010000 | \$ 9.96 | \$ 10.00 | 0.00630 | 0.07506 | \$ 1.03 | \$ 0.01 | \$ (1.02) | \$ 1.04 | \$ 2.38 | -0.9902913 |

Table 6.1: Excel Spreadsheet showing the call option pricing process using the Black-Scholes Formula

| Name of The Company | Stock Ticker | Symbol | Current Price of the Stock | Strike Price | Drift Parameter | Volatility Parameter | Calculated Model Option Price (MOP) | Observed Option Price (OOP) | Pricing Error (PE) | Squared PE | PE minus Mean of PE | Percentage Pricing Error (PPE) |
|---------------------------------------|--------------|---------------------|----------------------------|--------------|-----------------|----------------------|-------------------------------------|-----------------------------|--------------------|------------|---------------------|--------------------------------|
| Apple Inc. | AAPL | AAPL110820C00345000 | \$ 337.41 | \$ 345.00 | 0.01213 | 0.04761 | \$ 47.66 | \$ 20.02 | \$ (27.64) | \$ 763.97 | \$ 23.01 | -0.57994125 |
| Accenture plc | ACN | ACN110820C00057500 | \$ 56.91 | \$ 57.50 | 0.00470 | 0.03542 | \$ 2.26 | \$ 0.05 | \$ (2.21) | \$ 4.88 | \$ 2.42 | -0.978787611 |
| AT&T Inc. | T | T110820C00032000 | \$ 30.86 | \$ 32.00 | 0.00222 | 0.01941 | \$ 0.66 | \$ 0.01 | \$ (0.65) | \$ 0.42 | \$ 3.98 | -0.98484848 |
| Barnes & Noble, Inc. Common Sto | BKS | BKS110820C00020000 | \$ 19.56 | \$ 20.00 | 0.00027 | 0.07674 | \$ 1.58 | \$ 0.05 | \$ (1.53) | \$ 2.34 | \$ 3.10 | -0.96835443 |
| Best Buy Co., Inc. Common Stock | BBY | BBY110820C00032000 | \$ 31.44 | \$ 32.00 | 0.00712 | 0.06267 | \$ 3.52 | \$ 0.01 | \$ (3.51) | \$ 12.32 | \$ 1.12 | -0.99715909 |
| Capital One Financial Corp. | COF | COF110820C00055000 | \$ 53.97 | \$ 55.00 | 0.00142 | 0.12338 | \$ 6.92 | \$ 0.02 | \$ (6.90) | \$ 47.61 | \$ 3.27 | -0.99710983 |
| CIGNA Corporation | CI | CI110820C00050000 | \$ 49.47 | \$ 50.00 | 0.01009 | 0.08641 | \$ 7.80 | \$ 0.06 | \$ (7.74) | \$ 59.91 | \$ 3.11 | -0.99230769 |
| Cisco Systems, Inc. | CSCO | CSCO110820C00017000 | \$ 16.40 | \$ 17.00 | 0.00369 | 0.04347 | \$ 0.96 | \$ 0.01 | \$ (0.95) | \$ 0.90 | \$ 3.68 | -0.98958333 |
| CITI Group, INC | C | C110820C00042000 | \$ 40.96 | \$ 42.00 | -0.00731 | 0.18233 | \$ 5.71 | \$ 0.01 | \$ (5.70) | \$ 32.49 | \$ 1.07 | -0.99824869 |
| CVS Caremark Corporation | CVS | CVS110820C00040000 | \$ 38.67 | \$ 40.00 | 0.00081 | 0.04332 | \$ 1.67 | \$ 0.03 | \$ (1.64) | \$ 2.69 | \$ 2.99 | -0.98203593 |
| Dell Inc. | DELL | DELL110820C00016000 | \$ 15.79 | \$ 16.00 | 0.00261 | 0.05344 | \$ 1.17 | \$ 0.01 | \$ (1.16) | \$ 1.35 | \$ 3.47 | -0.99145299 |
| Delphi Financial Group, Inc | DFG | DFG110820C00030000 | \$ 28.72 | \$ 30.00 | 0.00847 | 0.09136 | \$ 3.78 | \$ 0.05 | \$ (3.73) | \$ 13.91 | \$ 0.90 | -0.998677249 |
| Expedia, Inc. | EXPE | EXPE110820C00028000 | \$ 27.69 | \$ 28.00 | 0.01235 | 0.07648 | \$ 4.67 | \$ 0.85 | \$ (3.82) | \$ 14.59 | \$ 0.81 | -0.81798715 |
| FedEX Corporation Common Stock | FDX | FDX110820C00095000 | \$ 91.76 | \$ 95.00 | 0.00339 | 0.05480 | \$ 6.34 | \$ 0.03 | \$ (6.31) | \$ 39.82 | \$ 1.68 | -0.99526814 |
| General Electric Company Common | GE | GE110820C00020000 | \$ 19.28 | \$ 20.00 | 0.00105 | 0.07061 | \$ 1.39 | \$ 0.01 | \$ (1.38) | \$ 1.90 | \$ 3.25 | -0.99280576 |
| Hartford Financial Services Group INC | HIG | HIG110820C00027000 | \$ 26.53 | \$ 27.00 | 0.00767 | 0.18567 | \$ 5.25 | \$ 0.02 | \$ (5.23) | \$ 27.35 | \$ 0.60 | -0.99619048 |
| Hewlett-Packard Company Common | HPQ | HPQ110820C00038000 | \$ 36.83 | \$ 38.00 | 0.00375 | 0.04293 | \$ 2.24 | \$ 0.01 | \$ (2.23) | \$ 4.97 | \$ 2.40 | -0.99553571 |
| Hitachi, Ltd. | HIT | HIT110820C00060000 | \$ 55.94 | \$ 60.00 | 0.00024 | 0.05075 | \$ 2.00 | \$ 0.70 | \$ (1.30) | \$ 1.69 | \$ 3.33 | -0.65 |
| HSBC Holdings PLC (ADR) | HBC | HBC110820C00052500 | \$ 51.71 | \$ 52.50 | 0.00129 | 0.06553 | \$ 4.06 | \$ 0.01 | \$ (4.05) | \$ 16.40 | \$ 0.58 | -0.99753695 |
| Humana Inc. | HUM | HUM110820C00082500 | \$ 79.15 | \$ 82.50 | 0.00640 | 0.08179 | \$ 8.74 | \$ 0.19 | \$ (8.55) | \$ 73.10 | \$ 3.92 | -0.97825087 |
| Intel Corporation | INTC | INTC110820C00023000 | \$ 22.21 | \$ 23.00 | 0.00510 | 0.04381 | \$ 1.53 | \$ 0.01 | \$ (1.52) | \$ 2.31 | \$ 3.11 | -0.99346405 |
| International Business Machines | IBM | IBM110820C00017000 | \$ 167.50 | \$ 170.00 | 0.00615 | 0.03130 | \$ 12.83 | \$ 0.03 | \$ (12.80) | \$ 163.84 | \$ 8.17 | -0.99766173 |
| ITT Educational Services Inc. | ESI | ESI110820C00072500 | \$ 70.87 | \$ 72.50 | -0.00283 | 0.06687 | \$ 4.07 | \$ 0.08 | \$ (3.99) | \$ 15.92 | \$ 0.64 | -0.98034398 |
| J.C.Penney Company, Inc. | JCP | JCP110820C00037000 | \$ 35.79 | \$ 37.00 | -0.00097 | 0.05896 | \$ 1.91 | \$ 0.07 | \$ (1.84) | \$ 3.39 | \$ 2.79 | -0.96335079 |
| John Wiley & Sons Inc. | JW-A | JWA110820C00050000 | \$ 51.62 | \$ 50.00 | 0.00313 | 0.05264 | \$ 5.26 | \$ 0.10 | \$ (5.16) | \$ 26.63 | \$ 0.53 | -0.98098859 |
| JP Morgan Chase & Co. Common St | JPM | JPM110820C00043000 | \$ 42.53 | \$ 43.00 | 0.00176 | 0.09600 | \$ 4.74 | \$ 0.02 | \$ (4.72) | \$ 22.28 | \$ 0.09 | -0.99578059 |
| Marriott International | MAR | MAR110820C00039000 | \$ 37.86 | \$ 39.00 | 0.00904 | 0.06549 | \$ 4.60 | \$ 0.05 | \$ (4.55) | \$ 20.70 | \$ 0.08 | -0.98913043 |
| Mastercard Incorporated | MA | MA110820C00029000 | \$ 285.24 | \$ 290.00 | 0.00549 | 0.05553 | \$ 26.67 | \$ 20.00 | \$ (6.67) | \$ 44.49 | \$ 2.04 | -0.25009374 |
| McDonald's Corporation | MCD | MCD110820C000825000 | \$ 81.62 | \$ 82.50 | 0.00398 | 0.02426 | \$ 4.34 | \$ 5.55 | \$ 1.21 | \$ 1.46 | \$ 5.84 | 0.278801843 |
| Microsoft Corporation | MSFT | MSFT110820C00025000 | \$ 24.76 | \$ 25.00 | 0.00322 | 0.04040 | \$ 1.64 | \$ 0.02 | \$ (1.62) | \$ 2.62 | \$ 3.01 | -0.98780488 |
| Morgan Stanley | MS | MS110820C00025000 | \$ 24.02 | \$ 25.00 | 0.00812 | 0.09089 | \$ 3.11 | \$ 0.01 | \$ (3.10) | \$ 9.61 | \$ 1.53 | -0.99678457 |
| Motorola Solutions Inc | MSI | MSI110820C00049000 | \$ 47.58 | \$ 49.00 | 0.00678 | 0.069581 | \$ 5.16 | \$ 0.45 | \$ (4.71) | \$ 22.18 | \$ 0.08 | -0.9127907 |
| Nokia Corporation | NOK | NOK110820C00009000 | \$ 8.20 | \$ 9.00 | -0.00085 | 0.05810 | \$ 0.26 | \$ 0.02 | \$ (0.24) | \$ 0.06 | \$ 4.39 | -0.92307692 |
| Oracle Corporation | ORCL | ORCL110820C00034000 | \$ 33.64 | \$ 34.00 | 0.005351 | 0.038193 | \$ 2.63 | \$ 0.04 | \$ (2.59) | \$ 6.71 | \$ 2.04 | -0.98479087 |
| PNC Financial Services | PNC | PNC110820C00062500 | \$ 62.16 | \$ 62.50 | -0.00047 | 0.10651 | \$ 6.95 | \$ 0.05 | \$ (6.90) | \$ 47.61 | \$ 2.27 | -0.99280576 |
| Prudential Financial, Inc. | PRU | PRU110820C00065000 | \$ 63.17 | \$ 65.00 | 0.00796 | 0.12183 | \$ 9.96 | \$ 0.03 | \$ (9.93) | \$ 98.60 | \$ 5.30 | -0.99698795 |
| Siemens AG | SI | SI110820C00130000 | \$ 128.19 | \$ 130.00 | 0.00826 | 0.05575 | \$ 15.05 | \$ 0.06 | \$ (14.99) | \$ 224.70 | \$ 10.36 | -0.99601329 |
| Starbucks Corporation | SBUX | SBUX110820C00037000 | \$ 36.46 | \$ 37.00 | 0.01229 | 0.05230 | \$ 5.52 | \$ 0.03 | \$ (5.49) | \$ 30.14 | \$ 0.85 | -0.98456522 |
| Texas Instruments Incorporated | TXN | TXN110820C00035000 | \$ 34.57 | \$ 35.00 | 0.00681 | 0.04416 | \$ 2.05 | \$ 0.02 | \$ (2.03) | \$ 4.12 | \$ 2.60 | -0.9902439 |
| The Allstate Corporation | ALL | ALL110820C00032000 | \$ 31.24 | \$ 32.00 | 0.00200 | 0.06246 | \$ 2.34 | \$ 0.01 | \$ (2.33) | \$ 5.43 | \$ 2.30 | -0.99572665 |
| United Parcel Services, Inc. Com | UPS | UPS110820C00075000 | \$ 72.96 | \$ 75.00 | 0.00314 | 0.04256 | \$ 4.25 | \$ 0.01 | \$ (4.24) | \$ 17.98 | \$ 0.89 | -0.99764706 |
| Visa Inc | V | V110820C00080000 | \$ 79.80 | \$ 80.00 | 0.00369 | 0.04907 | \$ 6.63 | \$ 0.95 | \$ (5.68) | \$ 32.26 | \$ 1.05 | -0.85671192 |
| Waddell & Reed Financial, Inc | WDR | WDR110820C00040000 | \$ 38.11 | \$ 40.00 | 0.01079 | 0.07665 | \$ 5.05 | \$ 0.25 | \$ (4.80) | \$ 23.04 | \$ 0.17 | -0.95049505 |
| Wal-Mart Stores, Inc. Common St | WMT | WMT110820C00055000 | \$ 54.70 | \$ 55.00 | 0.00081 | 0.02730 | \$ 2.06 | \$ 0.02 | \$ (2.04) | \$ 4.16 | \$ 2.59 | -0.99029126 |
| Xerox Corporation | XRX | XRX110820C00010000 | \$ 9.96 | \$ 10.00 | 0.00630 | 0.07506 | \$ 1.24 | \$ 0.01 | \$ (1.23) | \$ 1.51 | \$ 3.40 | -0.99193548 |

Table 6.2: Excel Spreadsheet showing the call option pricing process using the Monte-Carlo Simulation Model

Statistical Comparison of the Two Models

The significance of one model over the other is studied using three statistical tools as described below.

1. Visual Observation of Scattered Plots

To have a scattered plot analysis and better visual understanding of the two models, I calculated the Percentage Option Pricing Error (PPE). This is the deviation of the Observed

Option Price (OOP) from the calculated Model Option Price (MOP) for the forty five stock prices under consideration. The values for PPE using BS and MCS are indicated in Table 5.2 and Table 5.3 respectively. And the values were calculated using the formula

$$PPE_i = \frac{OOP_i - MOP_i}{MOP_i}$$

The values associated with PPE were plotted against the Strike Price, Volatility, MOP and OOP using Minitab. Figure 6.1 shows the scatter Plot of the above parameters using the Black-Scholes Formula while Figure 6.2 is using the MCS model. Visual comparison of the two plots indicates that both models are equally significant to the call option pricing process as the PPE plots against the four parameters look similar in both models. Even though there are some scattered points (outliers) for the plots in Figure 6.2, a general conclusion that both models are significant to the option pricing process can be reached.

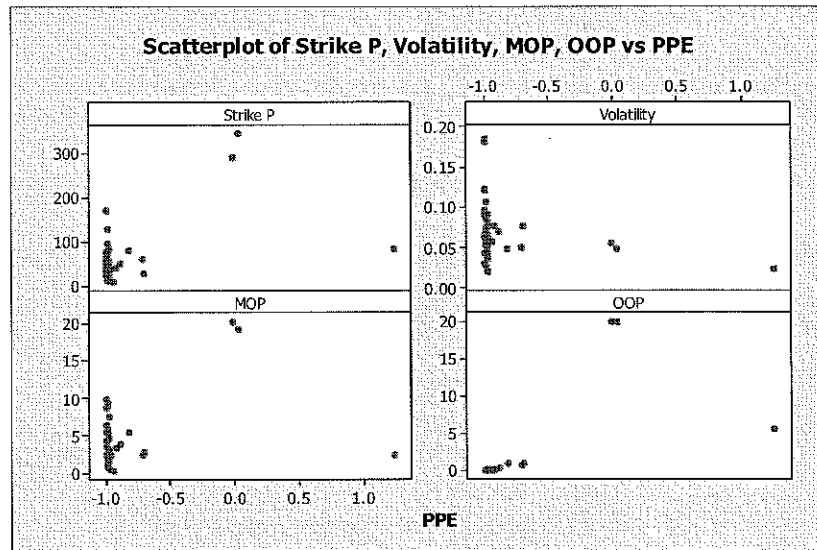


Figure 6.1: A scatter plot of Strike Price, Volatility, MOP, OOP vs. PPE with the Black-Scholes Model and using Minitab.

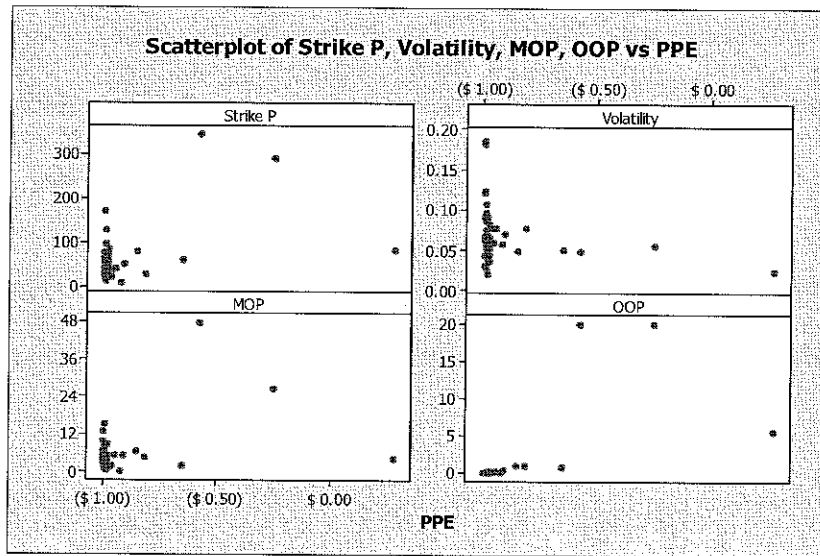


Figure 6.2: A scatter plot of Strike Price, Volatility, MOP, OOP vs. PPE with the MCS Model and using Minitab.

2. Standard Error of the Estimate (The Root of Mean Squared Error) and the Mean Absolute Deviation.

The second comparison tools used to see the significance of one model over the other are: the Root Mean Squared Error (RMSE) and the Mean Absolute Deviation (MAD). To calculate the RMSE and MAD, I considered the Observed Option Price (OOP), the Model Option Price (MOP) and the Price Error (PE).

The RMSE and MAD are calculated as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{45} (OOP_i - MOP_i)^2}{43}}$$

$$MAD = \frac{\sum_{i=1}^{45} \left| PE_i - \frac{\sum PE_i}{45} \right|}{45}$$

The values for RMSE and MAD for each model were calculated separately for each model and they are summarized in Table 6.4.

| Statistics Method | Black-Scholes Model | MCS Model |
|-------------------|---------------------|-----------|
| RMSE | 4.58 | 6.74 |
| MAD | 2.33 | 2.96 |

Table 6.4: RMSE and MAD values calculated for Black-Scholes Model and MCS Model.

Using the RMSE value, it appears that the BS model is more significant than the MCS model as we have a smaller value of RMSE for the BS model. But still the gap is not wide enough to conclude the significance BS over MCS.

Again using the MAD value, it appears that BS formula is more significant than the MCS model as we have a smaller value for the BS model. Also in here it will be difficult to conclude that the BS is a better model as the MAD difference is so small. At this point I would recommend using further statistical tool to study the significance of one over the other. Below is a discussion of the third statistical tool for studying the significance of the models.

3. Hypothesis Testing

The last comparison tool I used is a statistical test to compare the model prices against the observed price. To do so I considered the mean of Percentage Option Pricing Error from Black Scholes (PPEBS) and the mean of the Percentage Option Pricing Error obtained from MSC (PPEMCS). And I also defined two hypotheses: Null (H_0) and Alternative (H_a) Hypothesis

based on the assumption that “If one model is more significant than the other, then they wouldn’t have the same mean for PPE values”. The elements of the Statistical test are indicated below:

✦ Null Hypothesis

- H_0 : Mean of PPEBS = Mean of PPEMCS

✦ Alternative Hypothesis

- H_1 : Mean of PPEBS \neq Mean of PPEMCS

✦ Test Statistics : t-distribution

- $$t = \frac{\text{Mean of PPEBS} - \text{Mean of PPEMCS}}{\sqrt{\frac{\text{Var.of PPEBS} + \text{Var.of PPEMCS}}{2}} / \sqrt{n}}$$

✦ Rejection region:

A two –tail null hypothesis test is rejected if the t-statistics is larger than the corresponding critical value under a 95% confidence interval (that is $\frac{\alpha}{2} = 0.05$) with degrees of freedom 43. The Critical Value ($t_{\frac{\alpha}{2}}$) is 1.69.

- If $|t| < t_{\frac{\alpha}{2}}$, we conclude H_0 and reject H_a
- If $|t| > t_{\frac{\alpha}{2}}$, we conclude H_a and reject H_0

Table 6.5 summarizes the results of the two-tailed t-test run using Minitab.

| Model | N | Mean | Standard Deviation | t-value | p-value |
|-------|----|-----------|--------------------|---------|---------|
| BS | 45 | -0.874061 | 0.387628 | 0.65 | 0.516 |
| MCS | 45 | -0.917777 | 0.227044 | | |

Table 6.5: Results of the t-test for mean PE comparison of the BS and MCS Models using Minitab

At the significant level $\frac{\alpha}{2} = 0.05$, the observed value, is less than the critical value $t_{\frac{\alpha}{2}} = 1.69$.

There is a significant statistical evidence to accept the null hypothesis. This implies that both models are significant to the option pricing process.

General Conclusion:

There isn't enough statistical evidence to conclude that one model is more significant over the other for the option pricing process. It appears that the two models are not significantly different from each other.

Now at this point, you might ask one question: “Are both BS and MCS models significantly different from each other for Financial Companies? What about for Services Providers (Services) and Technology Oriented Companies (Technology)? Will the general conclusion made above works for the three categories of the companies we have?”

To check this, I used *Visual Observation of Scattered plots, RMSE and MAD values, and Hypothesis testing* for the 15 option price we have under the three categorized companies.

✦ *Visual Observation of Scattered plots*

✓ *Financial Companies*

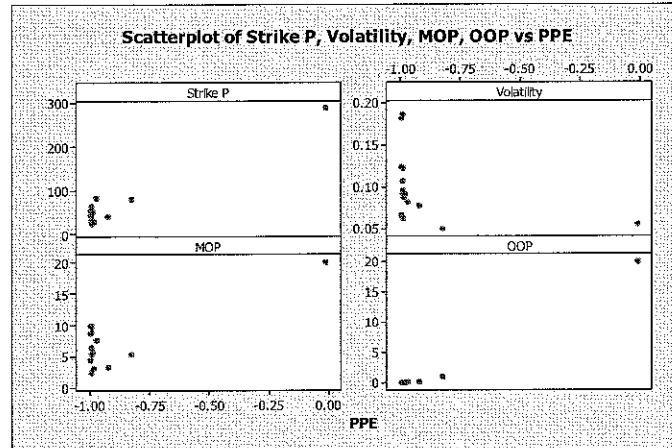


Figure 6.3: Scatter plot of Strike price, Volatility, MOP, OOP vs. PPE for Financial Companies using the BS Model

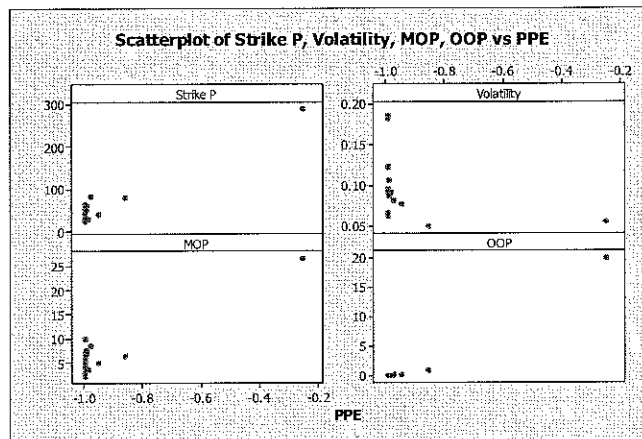


Figure 6.4: Scatter plot of Strike price, Volatility, MOP, OOP vs. PPE for Financial Companies using the MCS Model

Conclusion: Visual comparison of the above two plots indicate that both models are not significantly different from each other for Financial Companies as the PPE plots against the four parameters look similar in both models.

✓ Service Provider Companies (Services)

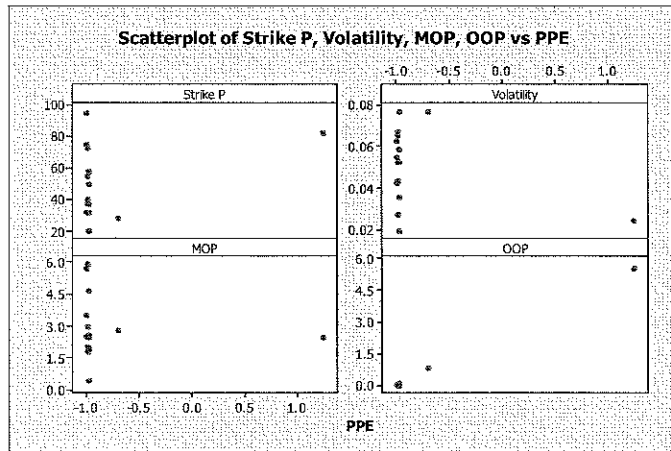


Figure 6.5: Scatter plot of Strike price, Volatility, MOP, OOP vs. PPE for Services Companies using the BS Model.

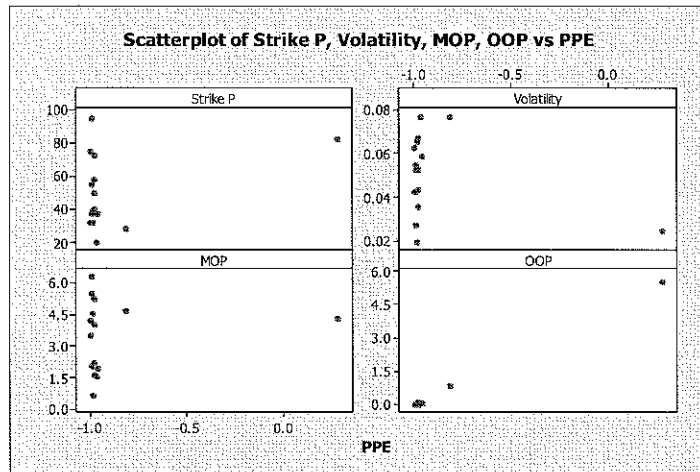


Figure 6.6: Scatter plot of Strike price, Volatility, MOP, OOP vs. PPE for Services Companies using the MCS Model.

Conclusion: The two plots we have above look similar to each other with very minor differences as such we conclude that the two models are not significantly different from each other in pricing the call option for the Service Companies.

✓ Technology Oriented Companies (Technology)

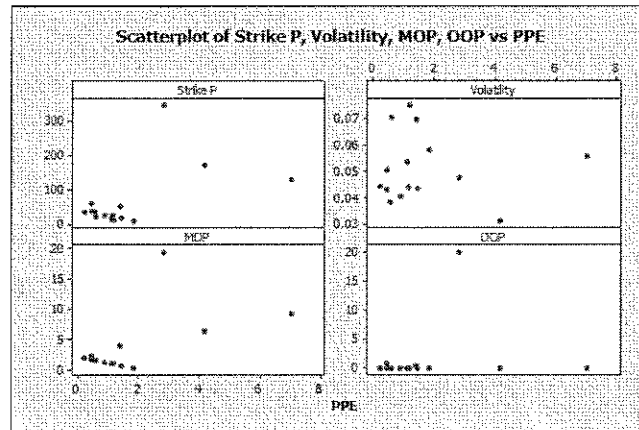


Figure 6.7: Scatter plot of Strike price, Volatility, MOP, OOP vs. PPE for Technology Companies using the BS Model.

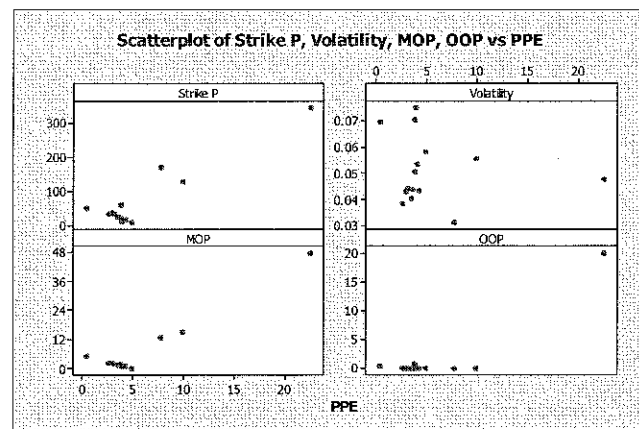


Figure 6.8: Scatter plot of Strike price, Volatility, MOP, OOP vs. PPE for Technology Companies using the MCS Model.

Conclusion: The visual observation for the two plots above indicates that the two models are not significantly different from each other for the Technology Companies.

✦ RMSE and MAD Value comparison:

Table 6.6 shows the RMSE and MAD values for each of the three companies considered under the two models.

| Statistics | Financial | | Services | | Technology | |
|-------------|-----------|-----------|----------|-----------|------------|-----------|
| | BS Model | MCS Model | BS Model | MCS Model | BS Model | MCS Model |
| RMSE | 6.705 | 6.529 | 3.473 | 3.897 | 3.498 | 9.619 |
| MAD | 2.463 | 1.637 | 1.357 | 1.688 | 1.683 | 5.354 |

Table 6.6: Comparison of the RMSE and MAD values for Financial, Services and Technology Companies using the BS and MCS models

Conclusion: For the Financial and Services Companies, there isn't a significant difference in the RMSE and MAD values obtained using the BS formula and MCS. As such I conclude that for these companies, both models are not significantly different from each other in pricing the call option for Financial and Services Companies. But for the Technology Companies, it appears that the BS Model is more significant than the MCS Model as the RMSE and MAD values obtained using the BS are considerably smaller than the ones obtained using the MCS model.

✓ Hypothesis Testing

Table 6.7, 6.8 and 6.9 summarizes the t-test run using Minitab for the Finance, Service and Technology companies considered in our study. The elements of the Statistical tests used are:

❖ Null Hypothesis

- H_0 : Mean of PPEBS = Mean of PPEMCS

❖ Alternative Hypothesis

- H_1 : Mean of PPEBS \neq Mean of PPEMCS

✓ Test Statistics : t-distribution

$$\text{❖ } t = \frac{\text{Mean of PPEBS} - \text{Mean of PPEMCS}}{\sqrt{\frac{\text{Var. of PPEBS} + \text{Var. of PPEMCS}}{2}} / \sqrt{n}}$$

✓ Rejection region :

A two –tail null hypothesis test is rejected if the t-statistics is larger than the corresponding critical value under a 95% confidence interval (that is $\frac{\alpha}{2} = 0.05$) with degrees of freedom 13. The Critical Value ($t_{\frac{\alpha}{2}}$) is 1.77.

- ❖ If $t < t_{\frac{\alpha}{2}}$, we conclude H_0 and reject H_a
- ❖ If $t > t_{\frac{\alpha}{2}}$, we conclude H_a and reject H_0

| Financial | | | | | |
|-----------|----|-----------|--------------------|---------|---------|
| Model | N | Mean | Standard Deviation | t-value | p-value |
| BS | 15 | -0.912085 | 0.253476 | 0.24 | 0.809 |
| MCS | 15 | -0.932121 | 0.192182 | | |

Table 6.7: Results of the t-test for mean PPE comparison of the BS and MCS Models using Minitab for Financial Companies

For the Financial Companies, at the significant level $\frac{\alpha}{2} = 0.05$, the observed value $t = 0.24$ is less than the critical value $t_{\frac{\alpha}{2}} = 1.77$. This indicates that there is a considerable statistical evidence to accept the null hypothesis. Hence the two models are not significantly different from each other for the Financial Companies.

| Services | | | | | |
|----------|----|-----------|--------------------|---------|---------|
| Model | N | Mean | Standard Deviation | t-value | p-value |
| BS | 15 | -0.816866 | 0.575692 | 0.42 | 0.675 |
| MCS | 15 | -0.889403 | 0.326168 | | |

Table 6.8: Results of the t-test for mean PE comparison of the BS and MCS Models using Minitab for Service Companies.

At a significant level of $\frac{\alpha}{2} = 0.05$, the observed value $t = 0.42$ is less than the critical value $t_{\frac{\alpha}{2}} = 1.77$ for the Service Companies. Also in this case we conclude the null hypothesis. Hence the two models of call option pricing process for the Service Companies are not significantly different from each other.

| Technology | | | | | |
|------------|----|-----------|--------------------|---------|---------|
| Model | N | Mean | Standard Deviation | t-value | p-value |
| BS | 15 | -0.893232 | 0.26670 | 0.50 | 0 |
| MCS | 15 | -0.931807 | 0.131902 | | |

Table 6.9: Results of the t-test for mean PE comparison of the BS and MCS Models using Minitab for Technology Companies

At a significant level of $\frac{\alpha}{2} = 0.05$, the observed value $t = 0.5$ is less than the critical value $t_{\frac{\alpha}{2}} = 1.77$ for the Technology Companies. This indicates that there is not a considerable statistical evidence to reject the null hypothesis. Hence the two models of call option pricing process for the Financial Companies are not significantly different from each other.

General Conclusion:

For all the three Companies considered, there isn't enough statistical evidence to conclude that one model is more significant over the other for the option pricing process. It appears that the two models are not significantly different from each other.

7. Conclusion

With the uncontrolled effect of the political and social situations to the current stock market, pricing a call option using a probabilistic model will not be an easy task. Accounting to the current economic situation of the world, a wide pricing error was observed between the modeled and actually observed option prices. Even though the option pricing models were undistinguishable from each other, a slight variation in the pricing error was observed. Over all, there was not significant statistical evidence supporting one model to be better than the other. Instead in future studies incorporating the political and social situations into the models might provide a better result. Studies also indicated that using different variance reduction techniques like Antithetic variates, Control variates, Moment Matching, Latin Hypercube Sampling, Importance Sampling, Conditional Monte Carlo and Quasi-Monte Carlo Simulation will also provide a better result.

As such for the Fellare's stock, I would recommend either using the Black Scholes or the MCS model to come up with the required figure.

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