

**AN ENROLLMENT RETENTION STUDY  
USING A MARKOV MODEL  
FOR A REGIONAL STATE UNIVERSITY CAMPUS  
IN TRANSITION**

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## **1. Introduction**

Enrollment retention rates are an accepted indicator of a university's success in providing quality degree programs and learning environments which will lead to a student's continued enrollment and timely graduation from the university campus.

This project will analyze enrollment data from five consecutive years from a state university regional campus, and construct an enrollment retention model using a Markov process. Due to the agreement of confidentiality, the name of the campus will not be revealed.

The model will be used to analyze enrollment retention rates for commonly overlooked segments of the student population, as well as the retention rates for specific degree programs, rather than just the retention rates for aggregate incoming freshmen.

This additional information will provide an opportunity for enrollment retention committees to focus on improvement for all classifications of students, rather than just the commonly observed freshmen, who are not necessarily representative.

The university from which the data has been collected is a regional campus of a state university which has, over the past 40 years, transitioned from an off-campus site, to a two-year community college, and recently, to an autonomous four-year-degree granting institution with graduate programs in planning and student housing under construction.

Retention figures tend to be used for comparison against national averages, and against rates of local, as well as similar peer institutions.

This project seeks to compare various retention rates within the schools of this campus for the purpose of gaining greater insight into areas where retention might be improved.

## **2. Literature Review**

### **2.1 Retention Rates**

There are two types of commonly accepted retention rates: the freshman-to-sophomore retention rate, and the six-year retention rate.

The freshman-to-sophomore retention rate is based on the number of first semester freshmen enrolling in a fall semester who then enroll as sophomores the following fall at the same campus. This retention rate is widely accepted as the key retention figure since students are most likely to drop out during their freshman year. The assumption is that students who progress to their sophomore year are less at risk for dropping out, and will likely go on to graduate from that campus.

The six-year retention rate is based on the number of first semester freshmen enrolling in a fall semester who go on to graduate from that campus, with a four-year degree, within a six year period. This retention rate seeks to measure timely graduation. In both instances, the retention rates apply only to full-time freshmen seeking a four-year degree.

Retention literature stresses that students who attend full-time and do not have outside employment have 70-75% graduation rate, nationally. Students who attend only part-time or who work full-time have an approximately 50% graduation rate. Further, only about 50% of students graduate from the institution where they began as freshmen. Of students whose college experience includes campus transfers, fewer than 10% progress to graduation. (Dworkin, 2005)

Retention rates are of interest not only to universities hopeful of maintaining or increasing enrollment, but act as a socio-economic indicator of well-being for the



community as a whole. It is, therefore, considered to be in the best public interest to maintain high retention rates.

## **2.2 Enrollment Retention Models**

Markov processes, regression models, and cohort flow models are three common techniques used for enrollment prediction. While all three methods are appropriate for the data available, the Markov model allows for a convenient table for viewing multiple states.

The Markov model makes use of readily available cross-sectional historical enrollment data and can be initiated at any point for which the data is available.

(Armacost and Wilson, 2002)

Markov models tend, however, to comprise a small number of states derived from aggregate student populations, specifically all students in all degree programs, or freshmen in all degree programs, rather than more specifically defined student classifications. (Kraft and Jarvis, 2005)

Further, Markov models have specifically appropriate uses. The Markov model assumes no trend in data, that the probability of flow from one state to the next will remain consistent for the life for the model. Yet, the university environment naturally evolves with changes to and additions of degree programs, changing student populations, and the growth of distance learning options. (UCF, 2005) The Markov model is best reserved for projections of no more than 10 years.

### **3. Purpose of the Project**

The retention rate definitions described above apply to full-time first semester freshmen enrolling in a fall semester, and only apply to those students pursuing a four-year degree.

A preliminary analysis of the data for the campus being studied indicates that at least one-third of the students are registered for 9 or few credit hours, with one-half of the students registered for fewer than 12 credit hours.

Slightly fewer than 25% of new students at this campus are first-time freshmen. About 10% of students are non-degree seeking students. This means that about 65% of the newly enrolled students are transfer students who are not represented in typical retention rate analysis.

Fewer than 30% of students are in a four-year degree program.

Finally, about half of all enrolled students are over the age of 25, with the average student age being approximately 29. Students over the age of 25 are classified as non-traditional or adult students. While available data does not include employment status, it would be reasonable to assume that many adults are employed at least part-time.

Regional campuses in general, and this campus, specifically, do not have the same student demographics as main campuses. Clearly, the normally accepted retention rate descriptions summarized in the previous section do not adequately account for the student demographics at this university campus. Therefore, a new model, using a Markov process, will be used to analyze available enrollment data with respect to appropriate rather than commonly accepted student classifications.

The one-year freshman-to-sophomore retention rate can be expanded for all student classifications, specifically the transfer students that comprise 65% of new students at this campus. Therefore, one-year retention rates will be determined for all classifications of students, as well as for traditional and non-traditional age groupings.

Similarly, a six-year retention model will be constructed for all students using a Markov process. Finally, a steady-state matrix will be developed in order to project the long-term graduation rate for students in more relevant groupings.

#### 4. Development of Model

The Markov model is based on an underlying stochastic process in which a system in one state,  $s_i$ , moves to a subsequent state,  $s_j$ . The states are commonly referred to as the Current state and the Next state. The act of moving from one state to the next is referred to as a *step* or *transition*.

The states are taken from a finite set of possible states  $S = \{s_1, s_2, \dots, s_n\}$  with each transition from a state  $s_i$  to a state  $s_j$  being defined by a transition probability  $p_{ji}$  of

occurring, and  $\sum_{j=1}^n p_{ji} = 1$  for  $i=1, \dots, n$ .

When the conditional probability of the Next state occurring, given the Current state and the sequence of state preceding it, is dependent only on the Current state, or independent of states previous to the Current state, the process is said to have the *memorylessness*, or Markov Property.

$P(X_{m+1} = s_j | X_0 = s_{i0}, X_1 = s_{i1}, X_2 = s_{i2} \dots, X_m = s_{im}) = P(X_{m+1} = s_j | X_m = s_{im})$ , where  $x_m$  is the current state,  $x_{m+1}$  is the next state, and  $1 \leq i_0, i_1, im \leq n$ .

A stochastic process with the Markov Property can be called a Markov Process or Markov Chain.

For the retention model, the transition probabilities,  $p_{ij}$ , will be obtained from the relative frequencies of students in semester number, or Classification Index  $i$  moving to Classification Index  $j$  in the period of one semester. Since only the Classification Index of the Current semester, and the Classification Index of the Next semester will be noted, the probability of moving from any Current state or semester to any Next state or semester will be independent of states previous to the Current state. This independence from previous states meets the criteria for the Markov Property, so this model can be justifiably called a Markov Process.

#### 4.1 Transition and Initial State Matrices

The Markov Process relies on two matrices: a transition matrix, and an initial state matrix.

The transition matrix is an  $n$ -by- $n$  matrix containing the transition probabilities,  $p_{ij}$ , for moving from the Current state in the process to the Next state. In this model, the Current states will be indicated by the matrix columns, and the Next states by the matrix rows. For the transition matrix  $T$ , each cell contains the following transition probability:

$$a_{ji} = p_{ji} = \Pr[s_j | s_i]$$

where  $j$  indicates the row, or Next state, and  $i$  indicates the column, or Current state, and  $1 \leq i, j \leq n$ .

The following is a simplified version of the transition matrix for this model.

		Current State				
		Freshman	Sophomore	Junior	Senior	
$T =$	Next State	Freshman	$p_{11}$	$p_{12}$	$p_{13}$	$p_{14}$
		Sophomore	$p_{21}$	$p_{22}$	$p_{23}$	$p_{24}$
		Junior	$p_{31}$	$p_{32}$	$p_{33}$	$p_{34}$
		Senior	$p_{41}$	$p_{42}$	$p_{43}$	$p_{44}$

The purpose of the transition matrix is to represent the probability of movement between states in a single time period. In this case, with what probability does a student achieve a particular Classification Index by the end of the Current semester?

The possible states into which a student could be classified are determined by the logical sorting criteria described below. The most basic states will include Classification Index (semester number), academic standing, and assumptions regarding non-enrollment.

A student may become non-enrolled through graduation, by being academically dropped by the university, or by leaving the university voluntarily without graduating (Lost.)

Initially, there will be 10 classification indexes, 2 levels of academic standing, and 3 reasons for leaving the university. All reasonable combinations of these factors will produce 23 individual states into which student data may be grouped, thus, necessitating a 23-by-23 transition matrix. The Table 4.1.1 lists these states and provides a description of each.

**Table 4.1.1**  
**States by CI and Standing**

STATE		DESCRIPTION	
ND	G	non-degree seeking,	good academic standing
ND	P	non-degree seeking,	academic probation
FF	G	first semester freshman,	good academic standing
FF	P	first semester freshman,	academic probation
1	G	1 - 15 credit hours,	good academic standing
1	P	1 - 15 credit hours,	academic probation
2	G	16 - 30 credit hours,	good academic standing
2	P	16 - 30 credit hours,	academic probation
3	G	31 - 45 credit hours,	good academic standing
3	P	31 - 45 credit hours,	academic probation
4	G	46 - 60 credit hours,	good academic standing
4	P	46 - 60 credit hours,	academic probation
5	G	61 - 75 credit hours,	good academic standing
5	P	61 - 75 credit hours,	academic probation
6	G	76 - 90 credit hours,	good academic standing
6	P	76 - 90 credit hours,	academic probation
7	G	91 - 105 credit hours,	good academic standing
7	P	91 - 105 credit hours,	academic probation
8	G	106+ credit hours,	good academic standing
8	P	106+ credit hours,	academic probation
X	D	no longer enrolled,	academically dropped
X	GR	no longer enrolled,	graduated
X	L	no longer enrolled,	lost

The data for students who are enrolled in the second, but not the first, of two consecutive semesters will be used to determine the initial state matrix.

Further sorting will be done by degree program, student age, and full- or part-time status.

The initial state matrix for the retention model will be a column matrix with a row for each state, or a 23-by-1 matrix. Each cell of the initial state matrix will contain the probability that a newly enrolled student will be classified in this state.

## 4.2 Application of the Markov Process

A Markov process is performed through matrix multiplication of the initial state matrix and the transition matrix, resulting in, for this application, a distribution of student states after one semester. In order to obtain a distribution matrix after many semester transitions, the transition matrix need only be applied the desired number of times.

Specifically,  $TX_0 = X_1$ , where  $T$  is the transition matrix,  $X_0$  is the initial state matrix, and  $X_1$  is the resulting distribution matrix after one transition.

Since the memorylessness property has been shown to be applicable, subsequent transitions, and subsequent distribution matrices can be obtained by using the previous distribution matrix as the current initial state matrix.

$$X_1 = TX_0$$

$$X_2 = TX_1 = T(TX_0) = T^2X_0$$

$$X_3 = TX_2 = T(T^2X_0) = T^3X_0$$

This process can be generalized to say that the distribution matrix, after  $m$  time periods or transitions is given by

$$X_m = T^m X_0$$

Since one objective of this model is to produce a six-year retention rate, the transition matrix will be applied (2 semesters) x (6 years) = 12 times, or

$$X_{12} = T^{12} X_0$$

### 4.3 The Steady-State Matrix

The distribution matrix,  $X_{12}$ , detailed in the previous section will provide a description of the student population as a whole after 12 semesters. For example, it might show that an initial state matrix representing a set of students of whom 7% are sophomores becomes, after 12 transitions, a distribution matrix of whom only 1% are sophomores. What this distribution matrix cannot describe is what has become of the original 7% of the students who were sophomores, or for that matter, what has become of any other specific group of students. It can only give a new distribution of the entire student population.

In order to discover longer term trends or graduation rates for specific groups of students, the steady state matrix is used. A matrix,  $X$ , is determined to be a steady state matrix if there is a transition matrix,  $T$ , such that  $XT = X$ .

The transition matrix,  $T$ , tends toward a fixed matrix,  $L$ , as the number of transitions,  $m$ , becomes large. In other words, it converges as  $m$  becomes large.

$L \approx T^m$  for large enough values of  $m$ . A Markov chain with this property is called *ergodic*.

For the retention model, the steady-state matrix,  $L$ , will describe the long term outcome for groups of students. Specifically, given enough time, students must either graduate, or leave the university, voluntarily or involuntarily. The steady-state matrix will give the probabilities for those outcomes by group. Since the steady-state matrix represents a long term trend, which may not actually be achieved in the university setting, in this context, it can be thought of as a “best case” scenario for current student demographics and movement.



The derivation of the steady-state matrix depends on the classification of the transition matrix. In this case, the transition matrix is an absorbing matrix. An absorbing stochastic matrix has the following properties:

1. There is at least one absorbing state, a state that once achieved is impossible to leave.
2. It is possible to move from any non-absorbing state to an absorbing state in one or more transitions.

There are three absorbing states in the retention transition matrix. These three states are the three conditions under which a student is no longer enrolled in the university, dropped, graduated, or lost. (Since only Current and Next enrollment periods are being considered, students who re-enroll after becoming non-enrolled are considered to be new students. This topic will be dealt with in more detail below.)

To further meet the criteria for classification as an absorbing Markov matrix, it can be shown, both from the transition matrix, and intuitively, that any absorbing state, any reason for leaving the university, can be reached from any other non-absorbing state or series of non-absorbing states. In other words, any student in any state can eventually graduate, become academically dropped, or become “lost.”

In order to develop the steady-state matrix for the absorbing Markov matrix, the transition matrix is required to be partitioned as follows:

		CURRENT	
		Absorbing	Nonabsorbing
NEXT	Absorbing	$I$	$S$
	Nonabsorbing	$O$	$R$

The left portion of the partitioned matrix contains the columns of the absorbing states, and the right portion contains the columns of the non-absorbing states.

Additionally, the top left portion will contain an identity matrix configured from the absorbing states, leaving the lower left a zero matrix. The right side of the matrix is partitioned as an effect of the partitioning of the absorbing states and represents the probability of movement among the non-absorbing states.

Given the partitioned absorbing transition matrix,  $A$ , following is the method for deriving the steady-state matrix.

$$A = \begin{bmatrix} I & S \\ 0 & R \end{bmatrix}$$

$$\text{steady-state } A = \begin{bmatrix} I & S(I - R)^{-1} \\ 0 & 0 \end{bmatrix}$$

$I$  is an  $m \times m$  identity matrix, where  $m$  is the number of absorbing states.  $S$  represents the probability of movement to absorbing states, while  $R$  shows the probability of movements among non-absorbing states.  $(I - R)^{-1}$  is the fundamental matrix, representing the number of times, starting from state  $i$ , the state  $j$  can be expected to be visited before absorption.

## 5. Technology

The data were made available from the campus registrar in the form of Excel spreadsheets. Sorting is done using the standard Excel sorting function.

The transitioning of the Markov model is done in Excel using the matrix multiplication function, `mmult(array1, array2)`.

This function is a tool for finding the ordinary matrix product of two arrays. The arrays may be, and in this case are, two-dimensional arrays, or matrices.

The product of  $m$ -by- $n$  Matrix A and  $n$ -by- $p$  Matrix B will be placed in  $m$ -by- $p$  Matrix C. Each element in Matrix C will be calculated as follows:

$$C_{ij} = A_{ik} \bullet B_{kj} = \sum_{k=1}^n a_{ik} b_{kj} = a_{i1} b_{1j} + a_{i2} b_{2j} + \dots + a_{in} b_{nj}$$

where  $1 \leq i \leq m$  and  $1 \leq j \leq p$ .

The Excel function `mmult(MatrixA, MatrixB)` is applied to each cell in the defined range for the destination Matrix C.

The calculation of the steady-state matrix is also done in Excel. In addition to using the `mmult(array1, array2)` function, it also requires the matrix inverse function `minverse(array)` where `array` is a square matrix.

The inverse of a matrix, A, is calculated by first joining it with an identity matrix of the same dimension to form the augmented matrix

$$[A | I]$$

which is then reduced, if possible, to the form

$$[I | B]$$

using Gauss-Jordan elimination. The matrix B is then the inverse of A, and  $AB=I$ .

## **6. Data, Assumptions, and Sorting**

The 10 most recent consecutive semesters' enrollment data (Fall 2000 - Spring 2005) were obtained from the university campus being studied. The complete set of data consisted of approximately 32,000 data points. Each data point contains data pertinent to an individual student's enrollment for a particular semester. The data used for sorting and determining states includes an anonymous student identification key, classification by credit hours earned, cumulative grade point average (GPA), number of hours registered, date of birth, and major code. Each data point does not represent a unique student. Several data points will represent a single student's enrollment over several semesters.

Data are included for all students enrolled in the given semesters. The data is a census, not a sample.

Only undergraduate student data are considered for this model. Graduate student data, as well as data for students earning post-baccalaureate certificates and endorsements were eliminated. This data set was identified using the degree code for each data point.

Also eliminated were data points for approximately 600 incoming students who had cumulative GPAs of 0.0. A GPA of 0.0 can be the result of a student's enrolling but then withdrawing before the end of the semester, by never attending and receiving an "unearned F," or by receiving a legitimate grade of F. No direct information in the data indicates the reason for the 0.0 GPA. Therefore, all such data points were eliminated based on the assumption that these data points likely represented withdrawals, but with the recognition that some legitimate data points might have been eliminated as well. These data points represented less than 2% of the total data points, but nearly 10% of

incoming students. Their inclusion would have made the first year retention rates artificially low.

Remaining data for use in this study were approximately 28,000 data points.

Following is a sample of the original data, where each row represents a data point, and the first ten rows represent a particular student.

**Table 6.0.1**  
**Sample of Original Data**

<b>Sess</b>	<b>Yr</b>	<b>Reg Hrs</b>	<b>Major1</b>	<b>CI</b>	<b>Birth Date</b>	<b>Cum Gpa</b>	<b>Name Key</b>
FA	2001	9	BAC	3	25-Jun-74	2.897	(name key)
SP	2002	9	BAC	6	25-Jun-74	2.969	(name key)
FA	2002	9	BAC	7	25-Jun-74	2.944	(name key)
SP	2003	9	BAC	8	25-Jun-74	2.949	(name key)
FA	2003	6	BAC	8	25-Jun-74	3.047	(name key)
SP	2004	6	BAC	8	25-Jun-74	3.044	(name key)
FA	2004	6	BAC	8	25-Jun-74	3.085	(name key)
SP	2005	6	BAC	8	25-Jun-74	3.102	(name key)
FA	2000	6	SGA	4	25-Jun-74	2.826	(name key)
SP	2001	6	SGA	5	25-Jun-74	2.84	(name key)
FA	2000	13	GPN	2	9-Oct-80	3.235	(name key)
SP	2001	12	NT	3	9-Oct-80	3.196	(name key)
FA	2001	8	NT	4	9-Oct-80	3.037	(name key)
SP	2002	7	NT	4	9-Oct-80	2.984	(name key)
FA	2002	7	NT	4	9-Oct-80	2.824	(name key)
FA	2003	6	NT	4	9-Oct-80	2.838	(name key)

The data were first sorted by name key by semester. This grouped all data points for the same student together, in chronological order. Then each student key was tested using an Excel IF function, flagging each data point that indicated the first appearance of a student name key. The flagged data points for semesters after Fall 2000 were marked as new incoming students. Data for new incoming students was used to calculate first year retention rates and the initial state matrix.

Each data point includes data for the Current semester. Each data point needed to be extended to include data for the Next semester as well. Since the data was chronologically sorted, the Next semester data point was on the line below. This second line was appended to the line above it using a simple Excel macro that copied the second line, and pasted it to cells at the end of the first line. The flagging mentioned above acted as a stopping mechanism for the macro in order to prevent two different students' data from being erroneously combined.

At this time, major codes were also used to include columns for major, school, and type of degree.

A sample of the extended data points follows in Table 6.0.2.

## **6.1 Sorting Criteria**

Student data was sorted by Classification Index, levels 1 through 8, which is determined by number of credit hours earned. A Classification Index of 1 indicates 1 to 15 credit hours earned, and is commonly thought of as first semester freshman; a Classification Index of 2 indicates 15 to 30 credit hours earned for a second semester freshman, and so forth. Special classifications include FF for first-time freshmen and ND for non-degree seeking students.

Data will also be sorted by cumulative GPA in order to further group students by academic standing. Students will be determined to be in good academic standing or on academic probation based on the criteria in the university's student handbook.

**Table 6.0.2  
Extended Data**

Name Key	CURRENT											NEXT										
	Sess	Yr	Reg Hrs	Major1	CI	Birth Date	Cum Gpa	Standing	Major	School	Degree	Sess	Yr	Reg Hrs	Major1	CI	Birth Date	Cum Gpa	Standing	Major	School	Degree
1 (name key)	FA	2003	12	PBS	1	17-Jun-85	2.333	G	Pre-Behavioral Science	General Studies	Pre	SP	2004	12	PBS	1	17-Jun-85	1.429	P	Pre-Behavioral Science	General Studies	Pre
1 (name key)	FA	2004	9	BPR	FF	7-May-85	1.333	P	Pre-General Business	General Studies	Pre	SP	2005	12	BPR	1	7-May-85	0.857	P	Pre-General Business	General Studies	Pre
1 (name key)	FA	2000	6	GGG	1	30-Nov-71	2	G	Bridge	General Studies	Bridge	SP	2001	3	GGG	1	30-Nov-71	1	P	Bridge	General Studies	Bridge
1 (name key)	FA	2002	13	GLA	1	27-Mar-84	0	P	Pre-Liberal Arts	General Studies	Pre	SP	2003	12	GLA	1	27-Mar-84	0.6	P	Pre-Liberal Arts	General Studies	Pre
1 (name key)	FA	2002	12	GSE	1	12-Aug-83	1	P	Pre-Education	General Studies	Pre	SP	2003	12	GSE	1	12-Aug-83	1.4	P	Pre-Education	General Studies	Pre
1 (name key)	FA	2003	14	PAT	1	10-Jan-85	0.667	P	Pre-Architectural Technology	General Studies	Pre	SP	2004	12	PAT	1	10-Jan-85	1.333	P	Pre-Architectural Technology	General Studies	Pre
1 (name key)	FA	2003	10	ANO	1	26-Aug-84	0.3	P	No Option	Agriculture	Transfer	SP	2004	12	ANO	1	26-Aug-84	0.682	P	No Option	Agriculture	Transfer
1 (name key)	FA	2002	6	GPN	1	26-Apr-80	0	P	Pre-Nursing	General Studies	Pre	SP	2003	6	GPN	1	26-Apr-80	1	P	Pre-Nursing	General Studies	Pre
(name key)	SP	2003	6	GPN	1	26-Apr-80	1	P	Pre-Nursing	General Studies	Pre	FA	2003	6	PPN	1	26-Apr-80	0.333	P	Pre-Nursing	General Studies	Pre
1 (name key)	FA	2002	12	GSE	1	7-Dec-83	1	P	Pre-Education	General Studies	Pre	SP	2003	12	GSE	1	7-Dec-83	0.571	P	Pre-Education	General Studies	Pre
1 (name key)	FA	2001	9	GGG	1	17-Apr-82	0.667	P	Bridge	General Studies	Bridge	SP	2002	3	GGG	1	17-Apr-82	0.5	P	Bridge	General Studies	Bridge
1 (name key)	FA	2000	12	GGG	1	4-Apr-81	1	P	Bridge	General Studies	Bridge	SP	2001	12	GGG	1	4-Apr-81	0.2	P	Bridge	General Studies	Bridge
1 (name key)	FA	2003	9	UJC	1	8-Nov-81	2	G	Journalistic Communications	Liberal Arts	Transfer	SP	2004	12	UJC	1	8-Nov-81	1.286	P	Journalistic Communications	Liberal Arts	Transfer
1 (name key)	FA	2004	13	DGG	FF	3-Jan-86	0	P	Bridge	General Studies	Bridge	SP	2005	13	DGG	1	3-Jan-86	0	P	Bridge	General Studies	Bridge
(name key)	FA	2000	13	GGG	1	17-Oct-64	1.3	P	Bridge	General Studies	Bridge	SP	2001	9	GGG	1	17-Oct-64	1.188	P	Bridge	General Studies	Bridge

Academic standing is based on a sliding scale which becomes more stringent for higher classification indexes. For example, a student with CI 1 is placed on academic probation when the cumulative GPA falls below 1.5; a student with CI 8 is placed on academic probation if the cumulative GPA falls below 2.0.

**Table 6.1.1  
Standing Scale by CI**

Classification Index	Cumulative GPA
1	1.5
2	1.6
3	1.7
4	1.8
5	1.9
6	2.0
7	2.0
8	2.0

Student age was calculated using student's date of birth, and the date of the semester. Students' ages do increase appropriately along the data points. Age will be used to sort students into traditional (under 26 years) and non-traditional age groups.

Full- or part-time status was determined by the number of credit hours registered for each semester. Students taking fewer than 12 credit hours are considered to be part-time students.

Further sorting will be done by degree program groups. In addition to the traditional 2-year Associate degree and 4-year Bachelor degree, students are also sorted into the following additional groups.



Bridge: A developmental program for students with deficiencies in their educational backgrounds. There are specific standards that must be met before exiting the bridge program.

High School: High achieving high school students who meet university entrance requirements may take one course per semester while still in high school. This is an option particularly attractive to home schooled high school students. The retention committee would like to entice these highly motivated students to enroll as degree seeking students, so their retention rates beyond high school is noteworthy.

Pre: Students who do not have university admission deficiencies, but are meeting prerequisites for their majors are admitted to a Pre program. Requirements for progressing from the Pre program to the major program varies greatly among degree programs. For example, competitive programs, such as the nursing program, have higher admission standards than university admissions, and a limited number of openings. Students may never advance from the pre-nursing program. However, students in pre-liberal studies, one of the biggest programs on the campus, just have to progress beyond Classification Index 3. Later, it will be noted that the Pre group is the degree group that contains the highest percentage of data points.

Transfer: Students who currently attend this campus but plan to transfer to a different campus of the same university, or to a different university entirely are Transfer students. Students planning to transfer are typically interested in a program not provided at this campus, or are interested in a different university environment. They are treated separately for retention rates, since they have no plan to graduate from this campus. It is the hope of the retention committee to create programs and environments that might

attract these students to stay. Graduation rates from the campus under study will be noted for students whose original plans were to transfer.

Undeclared: or Undecided is a catch-all group for many students, who, for example, enroll during late admission and have no specific plans beyond enrolling, and students who are visiting for a semester from other campuses. This category could also include students who are taking courses after graduation from their degree program, but many of those data points were likely flagged as graduate students and eliminated during the initial sort of the data. The university discourages students from taking courses without a degree objective.

A final preliminary sort groups data points into categories of CI, academic standing, age, hours register, and degree group for both Current and Next semesters. The number of data points in each of these very specific categories was tallied and used to summarize each of the categories. A sample of the data points summarized by categories follows in Table 6.1.2.

To interpret this summarized data, it can be noted from the first line that 62 full-time, traditionally aged students in good academic standing in CI 1 seeking an Associate degree were in exactly the same state the following semester. However, by reading line 2, it can be noted that 5 students who began in the state described above changed degree programs in the Next semester to seek a Bachelor degree.

**Table 6.1.2  
Data Summarized by Category**

CI	Standing	CURRENT			Group	CI	Standing	NEXT		Freq
		Age	Reg Hrs	Group				Reg Hrs	Group	
1	G	T	F	Associates	1	G	F	Associates	62	
1	G	T	F	Associates	1	G	F	Bachelors	5	
1	G	T	F	Associates	1	G	F	Pre	3	
1	G	T	F	Associates	1	G	F	Transfer	3	
1	P	T	P	Pre	1	G	P	Associates	2	
1	P	T	P	Pre	1	G	P	Bachelors	2	
1	P	T	P	Pre	1	G	P	Bridge	6	
1	P	T	P	Pre	1	G	P	Pre	11	
1	P	T	P	Pre	1	G	P	Transfer	1	
1	P	T	P	Pre	1	G	P	Undeclared	1	
2	G	T	F	Associates	2	G	F	Associates	7	
2	G	T	F	Associates	2	G	F	Bachelors	9	
2	G	T	F	Associates	2	G	F	Bridge	2	
2	G	T	F	Associates	2	G	F	Pre	6	
2	G	T	F	Associates	2	G	F	Transfer	4	
2	G	T	F	Associates	2	G	F	Undeclared	3	
2	P	T	F	Pre	3	P	F	Pre	1	
2	P	T	F	Pre	3	P	F	Transfer	1	
2	P	T	F	Pre	3	P	F	Undeclared	1	

## **6.2 Assumptions and Anomalies**

### **6.2.1 Assumptions about Non-enrollment**

Students who are enrolled in the first of two consecutive semesters but not the second became non-enrolled because they had been academically dropped by the university, graduated, transferred to another university, or stopped attending college altogether. No direct information about these states is available in the data. Assumptions are made based on Classification Index and academic standing.

Students in good academic standing with a CI of 8, and students in two-year degree programs with a CI of 4 or higher were assumed to have graduated after the Current semester.

Students who were on academic probation in the Current semester and non-enrolled in the Next semester were considered to have been academically dropped by the university.

No distinction could be made between students who transferred to other universities and students who stopped attending college altogether. Students in good academic standing but with CIs that were not near graduation in the Current semester were simply given the classification (L)ost.

### **6.2.2 Assumptions about New Students**

Students who were enrolled for the second of two consecutive semesters, but not the first were considered to be newly enrolled students of any possible classification index. This includes freshmen, transfers from other campuses, and non-degree seeking students. These new students were simply given the appropriate classification index.

Students who might have attended this campus previous to the time period of the data were still considered to be transfer students with previous college credits. No distinction is made for re-admitted students.

Students who enrolled for one or more semesters, become non-enrolled for any period of time, and then enroll again within the time period of the data were still considered to be newly enrolled students upon re-enrollment. Commentary regarding this phenomenon follows in the next section.

Students may be admitted to this campus with Unqualified Admission, Provisional Admission, or Probationary Admission. No information is given in the data regarding admission status. A backward assumption is therefore made that the admission status was likely to be that same as the academic standing at the end of Current semester. New students are, for this model, classified as Probationary or in Good Standing based on the Current semester GPA.

An exception is made in the case of the Initial State matrix for first-time freshmen (FF) and non-degree seeking students since it is university policy to accept such students only in Good Standing. First-time freshmen and non-degree students will be classified for the transition matrix as described above.

### **6.2.3 Continuing But Not Continuous Enrollment**

While sorting data and classifying student states, it became apparent that students commonly enroll for a number of semesters, drop out for a number of semesters, and then re-enroll.

In keeping with the memorylessness property that is a requirement for a Markov model, any enrollment status previous to the Current semester must be disregarded. As

such, re-enrollment could not be a consideration for this model. However, this pattern of enrollment might cause the drop out rate to be artificially high if these continuing but not continuous students do go on to graduate.

Distinct patterns of enrollment and non-enrollment, for example, the students who enroll for consecutive Fall semesters, but none of the Spring semesters, do bring to mind one of the most prominent characteristics of the student demographics that is not quantified in this data. At the time that this data was gathered, all students commuted to campus, and with rare exception, lived with family. Family and work/financial obligations nearly universally affect enrollment patterns for individual students.

A method was sought, therefore, to consider the phenomenon of continuing but not continuous enrollment, while maintaining the memorylessness property. This was accomplished by reclassifying the data points with breaks in enrollment not as (L)ost, but as (C)ontinuing. The data points for periods of enrollment were classified with the normal CI, but were not additionally marked as newly enrolled students.

This procedure can be justified as memorylessness in the same way that academic standing can be used as a classification even though it is a product of a cumulative GPA. Academic standing and the classification of (C)ontinuing are still only dependent on the previous semester's movement.

Further restrictions could be placed on the classification of (C)ontinuing, specifically, requiring enrollment after two consecutive semesters of (C)ontinuing, but implementation of this classification demonstrated that none of this was necessary.

Reclassification of continuing but not continuous students revealed that less than 5% of data points previous classified as (L)ost, and fewer than 1% of all data points were

effected. Further, of the data points that were reclassified as (C)ontinuing, only about 5% went on to have continuing enrollment. The remainder just went on to become (L)ost a semester later.

The conclusion drawn from this necessary investigation was that students who become non-enrolled and then re-enroll do not exist in high enough numbers to justify special distinction in this model. Therefore, re-enrolled students will be classified as newly enrolled transfer students, as described in the previous section.

#### **6.2.4 Anomalies in Movement**

It might seem reasonable that the logical transition for students from one semester to the next would be either to remain in the Classification Index they were in, give or take academic performance, or move to the next classification index.

It can be observed in the Data Summarized by Category and later in the Transition Matrix that movement can take the form of leaps over one or more indexes, or even move in reverse.

In the first instance, the leap over classification indexes can be explained by transfer credits that have been added during the semester, or credits received through test out. It is the accumulation of credit hours that determines the Classification Index, so movement need not be sequential.

Additionally, while there is no direct information obtained from the data, backward movement in classification index can be explained by changes in major or transfers to other schools within the campus. Such a change might result in loss of credit hours and an apparent backward movement.

Also, in spite of specific guidelines for Classification Index assignment, the process can still be subjective, and errors can occur. A new student's initial Classification Index is often a "best guess" based on information the student reports, which may not be accurate. Further, the specific Classification Index label does not have an impact on a student's actual progress. An incorrect or out of date CI might not be caught and corrected for several semesters. Unusual movement does not occur with enough frequency to affect rates being calculated.

## **7. Descriptive Statistics**

Values from the Data Summarized by Category is now used to calculate descriptive statistics for the sorting categories of interest. These data are shown in Table 7.0.0.1.

An overview of the descriptive statistics for new students reveals that the new admission Classification Index with the highest probability is Classification Index 1 at 62%. This is the group commonly referred to as the First Semester Freshman, but should not be confused with Classification Index FF, the First Time Freshman. CI 1 freshmen are those being admitted to the university with some previous college credits. In fact, nearly 80% of new students in this data have previous college credits, with only 13% of new students being First Time Freshmen (FF) and 8% being non-degree seeking students (ND) whose histories are unknown.



**Table 7.0.1  
Descriptive Statistics by Category**

<b>Classification Index</b>	<b>New Students</b>		<b>All Students</b>	
FF	661	13%	788	3%
1	3,225	62%	8,932	31%
2	337	6%	4,527	16%
3	147	3%	3,529	12%
4	219	4%	5,087	18%
5	64	1%	1,121	4%
6	47	1%	1,042	4%
7	23	0%	848	3%
8	103	2%	1,309	5%
ND	397	8%	1,182	4%
<b>TOTAL</b>	<b>5,223</b>		<b>28,365</b>	

<b>Academic Standing</b>	<b>New Students</b>		<b>All Students</b>	
Good Standing	4,817	92%	26,420	93%
Probation	406	8%	1,945	7%
<b>TOTAL</b>	<b>5,223</b>		<b>28,365</b>	

<b>Age</b>	<b>New Students</b>		<b>All Students</b>	
Traditional (under 26)	3,660	70%	18,378	65%
Non-traditional	1,563	30%	9,987	35%
<b>TOTAL</b>	<b>5,223</b>		<b>28,365</b>	

<b>Hours Registered</b>	<b>New Students</b>		<b>All Students</b>	
Full-time	2,666	51%	13,671	48%
Part-time (under 12 cr hr)	2,557	49%	14,694	52%
<b>TOTAL</b>	<b>5,223</b>		<b>28,365</b>	

<b>Degree Grouping</b>	<b>New Students</b>		<b>All Students</b>	
Associates	447	9%	5,100	18%
Bachelors	952	18%	8,366	29%
Bridge	701	13%	2,462	9%
High School	131	3%	310	1%
Pre	1,970	38%	8,475	30%
Transfer	512	10%	2,224	8%
Undeclared	510	10%	1,428	5%
<b>TOTAL</b>	<b>5,223</b>		<b>28,365</b>	

The descriptive statistics for all students reveals that the majority of all students, 80%, are in Classification Indexes FF through 4, or freshmen and sophomores. Sixteen percent of all students are upper level students in CI 5 through 8, with 4% of all students being non-degree seeking. There is such a significant difference between the percentages of lower level students with 12% to 31% in each Classification Index compared to 3% to 5% in upper level CIs that it might first appear that the community college environment still exists, with the majority of students attending for only two years. However, it will shortly be shown that the highest percentage of students is not for those seeking two-year degrees.

A high percentage of both new and all students are in good academic standing (92% and 93%.) Approximately half of both new and all students are part-time students.

Seventy percent of new students are traditionally aged college students, under 26 years. Slightly fewer, 65% of all students are traditionally aged. This is a higher percentage than the initially stated projection that only half of the students were traditionally aged.

The degree grouping with the highest probability of classification by new students is the Pre program at 38%, with 30% of all students in the Pre program. A higher percentage of new students, 18%, are admitted to a Bachelor program than an Associate program, 9%. This remains consistent for all students, with 29% in a Bachelor program and 18% in an Associate program, each gaining a higher percentage as students move from Pre and Bridge programs. The higher percentage of students in Bachelor programs than Associate programs refutes the notion that the higher percentage of students in lower

level Classification Indexes is due to choice of degree program and earlier graduation from two-year programs.

## **8. First Year Retention**

The Data Summarized by Category was further sorted to include only data points for students who appeared in the second of two consecutive semesters. These data points were marked as New Students. New Students were not just First Time Freshmen, CI FF, or even First Semester Freshmen, CI 1, but any new student in any Classification Index. Further, to be considered a New Student, the student may enroll for the first time in either the Spring or Fall semester.

If a new student is enrolled one year after first enrollment, that data point is marked as Retained. If the new student is not enrolled one year later, that data point is marked as Lost.

An analysis of the first year retention rates reveals that less than half of all new students, 46%, are retained to the following year. Of all students classified as First Time Freshmen, CI FF, only 1% are retained. Of CI 1, First Semester Freshmen with previous college credit, only 19% are retained. Much higher levels of retention are achieved by CIs 2 through 8, second semester freshmen through graduating seniors, supporting the assertion that incoming freshmen are more at risk for dropping out, but it should be noted that the upper level students with higher retention rates are still new, incoming transfer students. They are not students who have progressed passed incoming freshmen status at this campus.

**Table 8.0.1  
First Year Retention**

	Retained		Lost	
All Students	2,706	44%	3,474	56%

Classification Index	Retained		Lost	
FF	6	1%	659	99%
1	436	19%	1,894	81%
2	1,385	91%	144	9%
3	437	82%	97	18%
4	221	65%	117	35%
5	44	51%	43	49%
6	45	58%	33	42%
7	27	57%	20	43%
8	47	76%	15	24%
ND	58	11%	452	89%

Academic Standing	Retained		Lost	
Good Standing	2,601	46%	3,003	54%
Probation	105	18%	471	82%

Age	Retained		Lost	
Traditional	1,664	41%	2,402	59%
Non-traditional	1,042	49%	1,072	51%

Hours Registered	Retained		Lost	
Full-time	1,414	49%	1,487	51%
Part-time	1,292	39%	1,987	61%

Degree Group	Retained		Lost	
Associates	490	72%	190	28%
Bachelors	776	61%	494	39%
Bridge	205	28%	535	72%
High School	6	4%	128	96%
Pre	859	40%	1,303	60%
Transfer	235	40%	355	60%
Undeclared	135	22%	469	78%

The categories of academic standing, age, and hours registered show a similar retention pattern as the All Students category: slightly fewer than 50% of incoming students are retained to the next year. Exceptions can be seen for students on academic probation, of whom only 18% are retained, and for part-time students, of whom only 39% are retained.

When analyzing first year retention by degree group, it can be shown that 72% of students in Associate degree programs, and 61% of students in Bachelor degree programs are retained. This contrasts significantly with Bridge students at 28% and Pre students at 40%. A possible explanation for this phenomenon is that new students able to be admitted directly into a degree program are likely more capable students, in general.

Students planning to Transfer also have a 40% first year retention rate, while Bridge students are retained at 28%. Undeclared students have a 22% retention rate, and High School students, only a 4% retention rate. These lower retention rates for the less established degree groupings are not surprising, but still have the effect of lowering the retention rates for aggregate categories like Classification Index and Hours Registered which do not distinguish between degree groups.

## **9. Application of the Model to the Data**

### **9.1 Initial State Matrix**

Data points for students who enrolled in the second of two consecutive semesters but not the first were marked as newly enrolled students. The frequencies of these data points by category were used to calculate probabilities for the initial state matrix. The initial state matrix for data sorted by CI and academic standing follows.

**Table 9.1.1**  
**Initial State by CI and Academic Standing**

$X_0 =$

ND	G	0.0975
ND	P	0.0000
FF	G	0.1111
FF	P	0.0000
1	G	0.5681
1	P	0.0533
2	G	0.0435
2	P	0.0039
3	G	0.0266
3	P	0.0015
4	G	0.0388
4	P	0.0024
5	G	0.0120
5	P	0.0010
6	G	0.0073
6	P	0.0008
7	G	0.0052
7	P	0.0002
8	G	0.0266
8	P	0.0002
X	D	0.0000
X	GR	0.0000
X	L	0.0000

The initial state matrix by CI and academic standing shows that the greatest percentage of incoming students are First Semester Freshmen in good standing. Note that the initial state matrix contains probabilities of 0.0000 for Non-degree (ND) and First-time Freshmen (FF) on academic probation as described above.

It should be noted that the non-enrolled states are also included in this initial state matrix. It may seem counterintuitive to include such states; however, they must be included in order for the initial state and the transition matrices to have compatible dimensions.

## 9.2 Transition Matrix

The frequencies obtained in the Data Summarized by category were further condensed in order to produce the transition matrix by CI and academic standing, Table 9.2.0.1. For example, in the Data Summarized by category, there were 73 current students of CI 1 in good standing who remained CI 1 students in good standing at the end of the next semester. There were a total of 184 current CI 1 students in good standing, so  $\frac{73}{184} \rightarrow 0.397$  is the probability of a current CI 1 student in good standing remaining a CI 1 student if good standing at the end of the next semester.

The transition matrix shows, for example, that a student in CI 1 in good academic standing has a 0.3341 chance of, in the next semester, progressing to CI 2 while remaining in good academic standing.

Further examination of the matrix reveals that there are apparently states that are impossible to achieve, states for which there is a probability of 0.0000. For example, the probability of progressing from CI 4 in probationary academic standing to CI 5 in either good or probationary standing appears to be 0.000. Intuitively, it is certainly possible to make the transition from CI 4 to CI 5, but based on this data, highly unlikely. Of the 28,000 data points used to create the transition matrix, there were either no data points demonstrating this transition, or so few that the probability contains no significant digits, to four decimal places.

**Table 9.2.1**  
**Transition Matrix by CI and Academic Standing**

$T =$

		CURRENT																							
		ND G	ND P	FF G	FF P	1 G	1 P	2 G	2 P	3 G	3 P	4 G	4 P	5 G	5 P	6 G	6 P	7 G	7 P	8 G	8 P	X D	X GR	X L	
NEXT	ND G	0.3856	0.0889	0	-	0.0005	-	0.0002	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
	ND P	0.0075	0.0222	0	-	0.0001	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
	FF G	0.0131	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
	FF P	0	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
	1 G	0.1251	0.0222	0.5183	0.1414	0.3970	0.1020	0.0081	0.0311	0.0047	-	0.0040	-	0.0072	0.0667	0.0068	-	0.0047	0.3333	0.0107	-	-	-	-	-
	1 P	0.0065	0	0.0384	0.2626	0.0220	0.2092	0.0002	0.0104	-	-	0.0143	-	-	-	-	-	-	-	-	-	-	-	-	-
	2 G	0.0056	0	0.1728	-	0.3341	0.0077	0.4069	0.0363	0.0105	-	0.0032	-	0.0126	0.0667	0.0098	-	0.0012	-	0.0038	-	-	-	-	-
	2 P	0	0	0.0122	-	0.0096	0.0242	0.0070	0.1244	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	3 G	0	0.0222	0	-	0.0196	0.0013	0.3812	-	0.2811	0.0337	0.0111	-	0.0045	-	0.0098	-	0.0095	-	0.0031	-	-	-	-	-
	3 P	0	0	0	-	-	0.0013	0.0061	0.0725	0.0058	0.1573	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	4 G	0.0009	0	0	-	0.0036	-	0.0318	-	0.4872	-	0.5294	0.1429	0.0325	0.0667	0.0254	0.1111	0.0213	-	0.0404	-	-	-	-	-
	4 P	0	0	0	-	-	-	0.0052	0.0047	0.0674	0.0018	0.2000	-	-	0.0667	-	-	-	-	-	-	-	-	-	-
	5 G	0.0009	0	0	-	0.0012	-	0.0024	-	0.0183	-	0.1423	-	0.2385	0.0667	0.0117	-	0.0071	-	0.0023	-	-	-	-	-
	5 P	0	0	0	-	0.0001	-	-	-	-	-	0.0006	-	-	0.0667	-	-	-	-	-	-	-	-	-	-
	6 G	0	0	0	-	0.0013	-	0.0026	-	0.0035	-	0.0374	-	0.4571	0.0667	0.2361	-	0.0095	-	0.0053	-	-	-	-	-
	6 P	0	0	0	-	-	-	-	-	-	-	0.0006	-	0.0036	-	-	0.1667	-	-	-	-	-	-	-	-
	7 G	0.0009	0	0	-	0.0003	-	0.0006	-	0.0009	-	0.0187	-	0.0560	-	0.4849	0.0556	0.2270	-	0.0069	-	-	-	-	-
	7 P	0.0000	0	0	-	-	-	-	-	-	-	0.0002	-	-	-	-	0.0556	0.0024	-	-	-	-	-	-	-
	8 G	0.0009	0	0	-	0.0001	-	0.0004	-	0.0003	-	0.0175	-	0.0108	-	0.0634	-	0.5366	0.3333	0.4256	-	-	-	-	-
	8 P	0	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0012	-	-	-	-	-	-	-
X D	0	0.8444	0	0.5960	-	0.6543	0.0002	0.7202	-	0.7416	-	0.6429	-	0.5333	-	0.6111	-	0.3333	-	1.0000	1.0000	-	-	-	
X GR	0	0	0	-	-	-	-	-	0.0006	-	0.1183	-	0.0126	-	0.0049	-	0.0095	-	0.5019	-	-	1.0000	-	-	
X L	0.4528	0	0.2583	-	0.2106	-	0.1526	-	0.1826	-	0.1149	-	0.1644	-	0.1473	-	0.1702	-	-	-	-	-	1.0000	-	
check	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	



Similarly, the transition matrix suggests that any student in CI 8 with probationary standing would have a 100% probability of being academically dropped by the university. Again, this is a phenomenon of very few data points for students in probationary standing in CI 8. Those data points that do exist are all for students who were, in fact, academically dropped.

### **9.3 The Six-Year Graduation Rate**

With the initial state matrix and transition matrix in place, the six-year graduation rate can be calculated by transitioning the matrices as described above. Recall that in order to move the initial state matrix through six years, the initial state matrix must be multiplied twelve times by the transition matrix.

The following exhibit will show each of the twelve resulting matrices, with the twelfth showing the final six-year distribution of students.

**Table 9.3.1**  
**Twelve Distribution Matrices for 6-Year Distribution**

Semester 1	
ND G	0.0379
ND P	0.0008
FF G	0.0013
FF P	0.0000
1 G	0.3021
1 P	0.0286
2 G	0.2286
2 P	0.0089
3 G	0.0360
3 P	0.0010
4 G	0.0393
4 P	0.0009
5 G	0.0100
5 P	0.0002
6 G	0.0098
6 P	0.0002
7 G	0.0066
7 P	0.0001
8 G	0.0157
8 P	0.0000
X D	0.0416
X GR	0.0182
X L	0.2124
check	1.0000

Semester 2	
ND G	0.0149
ND P	0.0003
FF G	0.0005
FF P	0.0000
1 G	0.1311
1 P	0.0131
2 G	0.1957
2 P	0.0063
3 G	0.1040
3 P	0.0024
4 G	0.0483
4 P	0.0005
5 G	0.0098
5 P	0.0001
6 G	0.0096
6 P	0.0001
7 G	0.0080
7 P	0.0000
8 G	0.0118
8 P	0.0000
X D	0.0690
X GR	0.0310
X L	0.3437
check	1.0000

Semester 3	
ND G	0.0059
ND P	0.0001
FF G	0.0002
FF P	0.0000
1 G	0.0583
1 P	0.0058
2 G	0.1254
2 P	0.0037
3 G	0.1073
3 P	0.0026
4 G	0.0842
4 P	0.0009
5 G	0.0119
5 P	0.0001
6 G	0.0097
6 P	0.0001
7 G	0.0082
7 P	0.0000
8 G	0.0110
8 P	0.0000
X D	0.0846
X GR	0.0429
X L	0.4369
check	1.0000

Semester 4	
ND G	0.0023
ND P	0.0001
FF G	0.0001
FF P	0.0000
1 G	0.0268
1 P	0.0026
2 G	0.0724
2 P	0.0020
3 G	0.0804
3 P	0.0021
4 G	0.1024
4 P	0.0010
5 G	0.0174
5 P	0.0001
6 G	0.0118
6 P	0.0001
7 G	0.0091
7 P	0.0000
8 G	0.0114
8 P	0.0000
X D	0.0939
X GR	0.0587
X L	0.5051
check	1.0000

Semester 5	
ND G	0.0009
ND P	0.0000
FF G	0.0000
FF P	0.0000
1 G	0.0131
1 P	0.0012
2 G	0.0401
2 P	0.0011
3 G	0.0523
3 P	0.0014
4 G	0.0975
4 P	0.0009
5 G	0.0206
5 P	0.0001
6 G	0.0152
6 P	0.0001
7 G	0.0109
7 P	0.0000
8 G	0.0125
8 P	0.0000
X D	0.0994
X GR	0.0770
X L	0.5555
check	1.0000

Semester 6	
ND G	0.0004
ND P	0.0000
FF G	0.0000
FF P	0.0000
1 G	0.0069
1 P	0.0006
2 G	0.0221
2 P	0.0006
3 G	0.0318
3 P	0.0008
4 G	0.0804
4 P	0.0007
5 G	0.0202
5 P	0.0001
6 G	0.0171
6 P	0.0002
7 G	0.0130
7 P	0.0001
8 G	0.0141
8 P	0.0000
X D	0.1028
X GR	0.0953
X L	0.5930
check	1.0000

Semester 7	
ND G	0.0002
ND P	0.0000
FF G	0.0000
FF P	0.0000
1 G	0.0040
1 P	0.0003
2 G	0.0124
2 P	0.0003
3 G	0.0188
3 P	0.0005
4 G	0.0608
4 P	0.0005
5 G	0.0172
5 P	0.0001
6 G	0.0166
6 P	0.0001
7 G	0.0140
7 P	0.0001
8 G	0.0157
8 P	0.0000
X D	0.1048
X GR	0.1124
X L	0.6211
check	1.0000

Semester 8	
ND G	0.0001
ND P	0.0000
FF G	0.0000
FF P	0.0000
1 G	0.0026
1 P	0.0002
2 G	0.0073
2 P	0.0002
3 G	0.0112
3 P	0.0003
4 G	0.0438
4 P	0.0003
5 G	0.0135
5 P	0.0000
6 G	0.0144
6 P	0.0001
7 G	0.0135
7 P	0.0001
8 G	0.0166
8 P	0.0000
X D	0.1061
X GR	0.1279
X L	0.6420
check	1.0000

Semester 9	
ND G	0.0000
ND P	0.0000
FF G	0.0000
FF P	0.0000
1 G	0.0018
1 P	0.0001
2 G	0.0045
2 P	0.0001
3 G	0.0068
3 P	0.0002
4 G	0.0307
4 P	0.0002
5 G	0.0100
5 P	0.0000
6 G	0.0115
6 P	0.0001
7 G	0.0118
7 P	0.0000
8 G	0.0161
8 P	0.0000
X D	0.1069
X GR	0.1418
X L	0.6574
check	1.0000

Semester 10	
ND G	0.0000
ND P	0.0000
FF G	0.0000
FF P	0.0000
1 G	0.0013
1 P	0.0001
2 G	0.0029
2 P	0.0001
3 G	0.0043
3 P	0.0001
4 G	0.0213
4 P	0.0001
5 G	0.0071
5 P	0.0000
6 G	0.0087
6 P	0.0001
7 G	0.0095
7 P	0.0000
8 G	0.0146
8 P	0.0000
X D	0.1074
X GR	0.1538
X L	0.6686
check	1.0000

Semester 11	
ND G	0.0000
ND P	0.0000
FF G	0.0000
FF P	0.0000
1 G	0.0010
1 P	0.0000
2 G	0.0020
2 P	0.0000
3 G	0.0028
3 P	0.0001
4 G	0.0148
4 P	0.0001
5 G	0.0050
5 P	0.0000
6 G	0.0063
6 P	0.0001
7 G	0.0073
7 P	0.0000
8 G	0.0123
8 P	0.0000
X D	0.1077
X GR	0.1639
X L	0.6766
check	1.0000

Semester 12	
ND G	0.0000
ND P	0.0000
FF G	0.0000
FF P	0.0000
1 G	0.0007
1 P	0.0000
2 G	0.0014
2 P	0.0000
3 G	0.0019
3 P	0.0000
4 G	0.0103
4 P	0.0001
5 G	0.0035
5 P	0.0000
6 G	0.0045
6 P	0.0000
7 G	0.0053
7 P	0.0000
8 G	0.0099
8 P	0.0000
X D	0.1080
X GR	0.1720
X L	0.6823
check	1.0000

Specifically, to obtain cell  $ij$  of matrix  $X_I$ , the distribution matrix at the end of the first semester, the following matrix algebra was used.  $T$  is the transition matrix, and  $X_0$  is the initial state matrix.

$$\begin{aligned} X_{1,ij} &= T_{ik} \bullet X_{0,kj} = \sum_{k=1}^n t_{ik} x_{0,kj} = t_{i1} x_{0,1j} + t_{i2} x_{0,2j} + \dots + t_{in} x_{0,nj} \\ &= \sum_{k=1}^{23} (0.3865)(0.0975) + (0.0889)(0.0000) + \dots + (0.0000)(0.0000) \\ &= 0.0379 \end{aligned}$$

The actual calculation was performed using the matrix multiplication function in Excel: `mmult(T, X0)`, where the matrix names are also defines range names. The result of the calculation was placed in range  $X_I$ . By analyzing the Semester 12 distribution matrix, it can be shown that, according to this model, 17.2% of the incoming students had graduated within 6 years. Of these same incoming students, 68.23% had left the university voluntarily without graduating, and 10.8% had been academically dropped.

These figures only account for 96.23% of the incoming students. The remaining 3.77% of the incoming students are still enrolled at the end of six years, in various states of Classification Index and Academic Standing.

#### 9.4 A Simpler Variation of the Model

The previous application of the model used states grouped by Classification Index and Academic Standing, resulting in 23 individual states into which students could move. However, in the final distribution matrix, Semester 12, the Academic Standing was irrelevant since 96.23% of students had become non-enrolled.

In this next application of the model, the states will be grouped by Classification Index alone, and Academic Standing will be omitted. This will reduce the number of states through which students can transition to 13, and reducing the transition matrix by almost half.

The next three exhibits will show the revised Initial State and Transition matrices, and the new twelve semester distributions matrices. As before the twelfth semester distribution matrix will show the final six-year distribution of students.

**Table 9.4.1**  
**Initial State Matrix by CI Only**

$Y_0 =$

ND	0.0975
FF	0.1111
1	0.6214
2	0.0474
3	0.0282
4	0.0411
5	0.0130
6	0.0081
7	0.0054
8	0.0268
X D	0
X GR	0
X L	0
check	1.0000

**Table 9.4.2**  
**Transition Matrix by CI Only**

$T =$

		CURRENT													
		ND	FF	1	2	3	4	5	6	7	8	X D	X GR	X L	
NEXT	ND	0.3817	-	0.0006	0.0002	-	-	-	-	-	-	-	-	-	-
	FF	0.0125	-	-	0.0002	-	-	-	-	-	-	-	-	-	-
	1	0.1272	0.5342	0.4090	0.0094	0.0045	0.0041	0.0080	0.0067	0.0059	0.0107	-	-	-	-
	2	0.0054	0.1577	0.3150	0.4052	0.0102	0.0031	0.0134	0.0096	0.0012	0.0038	-	-	-	-
	3	0.0009	-	0.0181	0.3765	0.2845	0.0110	0.0045	0.0096	0.0094	0.0030	-	-	-	-
	4	0.0009	-	0.0033	0.0309	0.4812	0.5286	0.0339	0.0268	0.0212	0.0403	-	-	-	-
	5	0.0009	-	0.0012	0.0023	0.0179	0.1410	0.2371	0.0115	0.0071	0.0023	-	-	-	-
	6	-	-	0.0012	0.0025	0.0034	0.0375	0.4554	0.2349	0.0094	0.0053	-	-	-	-
	7	0.0009	-	0.0002	0.0005	0.0009	0.0186	0.0553	0.4784	0.2285	0.0068	-	-	-	-
	8	0.0009	-	0.0001	0.0004	0.0003	0.0173	0.0107	0.0623	0.5371	0.4247	-	-	-	-
	X D	0.0341	0.0878	0.0602	0.0246	0.0187	0.0088	0.0071	0.0105	0.0012	0.0023	1.0000	-	-	-
X GR	-	-	-	-	0.0006	0.1167	0.0125	0.0048	0.0094	0.5008	-	1.0000	-	-	
X L	0.4346	0.2202	0.1913	0.1474	0.1780	0.1133	0.1622	0.1448	0.1696	-	-	-	1.0000	-	
check	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	

**Table 9.4.3  
Twelve Distribution Matrices for 6-Year Distribution  
by CI Only**

**Semester 1**

ND	0.0376
FF	0.0012
1	0.3272
2	0.2338
3	0.0379
4	0.0407
5	0.0105
6	0.0105
7	0.0071
8	0.0158
X D	0.0528
X GR	0.0185
X L	0.2065
check	1.0000

**Semester 2**

ND	0.0146
FF	0.0005
1	0.1421
2	0.1990
3	0.1054
4	0.0495
5	0.0101
6	0.0100
7	0.0083
8	0.0121
X D	0.0809
X GR	0.0314
X L	0.3359
check	1.0000

**Semester 3**

ND	0.0057
FF	0.0002
1	0.0631
2	0.1271
3	0.1083
4	0.0848
5	0.0121
6	0.0100
7	0.0085
8	0.0114
X D	0.0975
X GR	0.0436
X L	0.4278
check	1.0000

**Semester 4**

ND	0.0022
FF	0.0001
1	0.0290
2	0.0731
3	0.0810
4	0.1024
5	0.0173
6	0.0119
7	0.0092
8	0.0117
X D	0.1077
X GR	0.0595
X L	0.4948
check	1.0000

**Semester 5**

ND	0.0009
FF	0.0000
1	0.0141
2	0.0404
3	0.0525
4	0.0970
5	0.0204
6	0.0152
7	0.0109
8	0.0127
X D	0.1140
X GR	0.0777
X L	0.5442
check	1.0000

**Semester 6**

ND	0.0004
FF	0.0000
1	0.0074
2	0.0221
3	0.0318
4	0.0797
5	0.0198
6	0.0170
7	0.0128
8	0.0141
X D	0.1181
X GR	0.0959
X L	0.5810
check	1.0000

**Semester 7**

ND	0.0001
FF	0.0000
1	0.0043
2	0.0124
3	0.0188
4	0.0601
5	0.0169
6	0.0164
7	0.0138
8	0.0155
X D	0.1207
X GR	0.1127
X L	0.6083
check	1.0000

**Semester 8**

ND	0.0001
FF	0.0000
1	0.0027
2	0.0072
3	0.0112
4	0.0432
5	0.0132
6	0.0141
7	0.0132
8	0.0162
X D	0.1225
X GR	0.1279
X L	0.6286
check	1.0000

**Semester 9**

ND	0.0000
FF	0.0000
1	0.0019
2	0.0044
3	0.0068
4	0.0302
5	0.0097
6	0.0112
7	0.0114
8	0.0157
X D	0.1237
X GR	0.1414
X L	0.6435
check	1.0000

**Semester 10**

ND	0.0000
FF	0.0000
1	0.0013
2	0.0028
3	0.0043
4	0.0209
5	0.0069
6	0.0084
7	0.0092
8	0.0141
X D	0.1246
X GR	0.1531
X L	0.6543
check	1.0000

**Semester 11**

ND	0.0000
FF	0.0000
1	0.0010
2	0.0019
3	0.0028
4	0.0144
5	0.0049
6	0.0061
7	0.0070
8	0.0119
X D	0.1252
X GR	0.1628
X L	0.6620
check	1.0000

**Semester 12**

ND	0.0000
FF	0.0000
1	0.0007
2	0.0013
3	0.0019
4	0.0100
5	0.0034
6	0.0043
7	0.0051
8	0.0095
X D	0.1256
X GR	0.1706
X L	0.6675
check	1.0000

By analyzing the Semester 12 distribution matrix for the simplified CI Only model, it can be shown that the final six-year distribution is very similar to the result obtained by the CI and Academic Standing model.

According to this model, 17.06% of students had graduated within 6 years, 66.75% of students had left the university voluntarily without graduating, and 12.56% had been academically dropped by the university.

Again, this accounts for only 96.37% of the initial distribution of students. The remaining 3.63% of students are still enrolled in various states of Classification Index.

The simpler version of the model yields virtually the same result. Therefore, this would be the most appropriate version of the model to use since it would require less sorting and a smaller model.

## **10. Application of the Steady-State Model**

The steady-state model as described above will be applied to the data developed in the last section. The steady-state model will use the same transition matrix as the CI Only version of the model. An initial state matrix is not required for the steady-state model.

First, the distribution matrix developed above will be partitioned into absorbing and non-absorbing states. This is simple a rearrangement of rows and columns so that the absorbing states form the upper left quadrant of the partitioned matrix. This partition is an identity matrix, since the probability of remaining in an absorbing state is always 1. The upper right quadrant contains the probabilities of moving from a non-absorbing state, to an absorbing state. The lower right quadrant shows the probabilities of movement among the non-absorbing states. The probabilities in the two right quadrants are the



same for correlating row and column headings in the transitions matrix,  $T$ . The lower left quadrant shows the probability of movement from an absorbing state to a non-absorbing state. Since this is impossible, the lower left quadrant is a zero matrix. The complete partitioned transition matrix follows.

Additional exhibits will show the sequence of matrices that are required to develop the steady state matrix.

**Table 10.0.1  
Partitioned Transition Matrix by CI Only**

		CURRENT												
		X D	X GR	X L	ND	FF	1	2	3	4	5	6	7	8
NEXT	X D	1.0000	0	0	0.0341	0.0878	0.0602	0.0246	0.0187	0.0088	0.0071	0.0105	0.0012	0.0023
	X GR	0	1.0000	0	0	0	0	0	0.0006	0.1167	0.0125	0.0048	0.0094	0.5008
	X L	0	0	1.0000	0.4346	0.2202	0.1913	0.1474	0.1780	0.1133	0.1622	0.1448	0.1696	0
	ND	0	0	0	0.3817	0	0.0006	0.0002	0	0	0	0	0	0
	FF	0	0	0	0.0125	0	0	0.0002	0	0	0	0	0	0
	1	0	0	0	0.1272	0.5342	0.4090	0.0094	0.0045	0.0041	0.0080	0.0067	0.0059	0.0107
	2	0	0	0	0.0054	0.1577	0.3150	0.4052	0.0102	0.0031	0.0134	0.0096	0.0012	0.0038
	3	0	0	0	0.0009	0	0.0181	0.3765	0.2845	0.0110	0.0045	0.0096	0.0094	0.0030
	4	0	0	0	0.0009	0	0.0033	0.0309	0.4812	0.5286	0.0339	0.0268	0.0212	0.0403
	5	0	0	0	0.0009	0	0.0012	0.0023	0.0179	0.1410	0.2371	0.0115	0.0071	0.0023
	6	0	0	0	0	0	0.0012	0.0025	0.0034	0.0375	0.4554	0.2349	0.0094	0.0053
	7	0	0	0	0.0009	0	0.0002	0.0005	0.0009	0.0186	0.0553	0.4784	0.2285	0.0068
8	0	0	0	0.0009	0	0.0001	0.0004	0.0003	0.0173	0.0107	0.0623	0.5371	0.4247	
check		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

**Table 10.0.2**  
**Steady-State Calculation by CI Only**

**Identity Matrix**

1.0000	-	-	-	-	-	-	-	-	-
-	1.0000	-	-	-	-	-	-	-	-
-	-	1.0000	-	-	-	-	-	-	-
-	-	-	1.0000	-	-	-	-	-	-
-	-	-	-	1.0000	-	-	-	-	-
-	-	-	-	-	1.0000	-	-	-	-
-	-	-	-	-	-	1.0000	-	-	-
-	-	-	-	-	-	-	1.0000	-	-
-	-	-	-	-	-	-	-	1.0000	-
-	-	-	-	-	-	-	-	-	1.0000

**Partition R of Transition Matrix**

0.3817	-	0.0006	0.0002	-	-	-	-	-	-
0.0125	-	-	0.0002	-	-	-	-	-	-
0.1272	0.5342	0.4090	0.0094	0.0045	0.0041	0.0080	0.0067	0.0059	0.0107
0.0054	0.1577	0.3150	0.4052	0.0102	0.0031	0.0134	0.0096	0.0012	0.0038
0.0009	-	0.0181	0.3765	0.2845	0.0110	0.0045	0.0096	0.0094	0.0030
0.0009	-	0.0033	0.0309	0.4812	0.5286	0.0339	0.0268	0.0212	0.0403
0.0009	-	0.0012	0.0023	0.0179	0.1410	0.2371	0.0115	0.0071	0.0023
-	-	0.0012	0.0025	0.0034	0.0375	0.4554	0.2349	0.0094	0.0053
0.0009	-	0.0002	0.0005	0.0009	0.0186	0.0553	0.4784	0.2285	0.0068
0.0009	-	0.0001	0.0004	0.0003	0.0173	0.0107	0.0623	0.5371	0.4247

**(I – R) Calculation**

0.618	-	(0.001)	(0.000)	-	-	-	-	-	-
(0.013)	1.000	-	(0.000)	-	-	-	-	-	-
(0.127)	(0.534)	0.591	(0.009)	(0.005)	(0.004)	(0.008)	(0.007)	(0.006)	(0.011)
(0.005)	(0.158)	(0.315)	0.595	(0.010)	(0.003)	(0.013)	(0.010)	(0.001)	(0.004)
(0.001)	-	(0.018)	(0.376)	0.716	(0.011)	(0.004)	(0.010)	(0.009)	(0.003)
(0.001)	-	(0.003)	(0.031)	(0.481)	0.471	(0.034)	(0.027)	(0.021)	(0.040)
(0.001)	-	(0.001)	(0.002)	(0.018)	(0.141)	0.763	(0.012)	(0.007)	(0.002)
-	-	(0.001)	(0.002)	(0.003)	(0.037)	(0.455)	0.765	(0.009)	(0.005)
(0.001)	-	(0.000)	(0.001)	(0.001)	(0.019)	(0.055)	(0.478)	0.771	(0.007)
(0.001)	-	(0.000)	(0.000)	(0.000)	(0.017)	(0.011)	(0.062)	(0.537)	0.575

**Table 10.0.2 (continued)**  
 **$(I - R)^{-1}$  Inverse of  $(I - R)$  Matrix**

1.6178	0.0011	0.0019	0.0006	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000
0.0203	1.0002	0.0002	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3743	0.9294	1.7234	0.0550	0.0394	0.0388	0.0527	0.0471	0.0411	0.0363
0.2231	0.7730	0.9321	1.7434	0.0691	0.0535	0.0822	0.0602	0.0374	0.0342
0.1326	0.4421	0.5489	0.9440	1.4698	0.0785	0.0816	0.0723	0.0491	0.0313
0.1650	0.5361	0.6681	1.1358	1.5837	2.3098	0.3302	0.2879	0.2333	0.1969
0.0376	0.1159	0.1452	0.2429	0.3349	0.4390	1.3952	0.0873	0.0621	0.0439
0.0330	0.1030	0.1295	0.2140	0.2889	0.3819	0.8605	1.3914	0.0745	0.0493
0.0298	0.0874	0.1099	0.1819	0.2459	0.3280	0.6482	0.8850	1.3640	0.0545
0.0398	0.1119	0.1405	0.2331	0.3153	0.4251	0.7343	0.9872	1.2896	1.8011

**S Partition of Transition Matrix**

0.0341	0.0878	0.0602	0.0246	0.0187	0.0088	0.0071	0.0105	0.0012	0.0023
-	-	-	-	0.0006	0.1167	0.0125	0.0048	0.0094	0.5008
0.4346	0.2202	0.1913	0.1474	0.1780	0.1133	0.1622	0.1448	0.1696	-

**$S(I - R)^{-1}$  Calculation**

0.0896	0.1781	0.1457	0.0787	0.0520	0.0340	0.0311	0.0268	0.0122	0.0104
0.0402	0.1216	0.1521	0.2555	0.3514	0.4928	0.4339	0.5441	0.6870	0.9262
0.8703	0.7003	0.7022	0.6657	0.5966	0.4732	0.5350	0.4291	0.3008	0.0634

**Final Steady-State Partition**

		ND	FF	1	2	3	4	5	6	7	8
X	D	0.0896	0.1781	0.1457	0.0787	0.0520	0.0340	0.0311	0.0268	0.0122	0.0104
X	GR	0.0402	0.1216	0.1521	0.2555	0.3514	0.4928	0.4339	0.5441	0.6870	0.9262
X	L	0.8703	0.7003	0.7022	0.6657	0.5966	0.4732	0.5350	0.4291	0.3008	0.0634

The final steady-state partition in the previous exhibit shows the probability for students in each Classification index reaching the absorbing states of the transition matrix, that is, graduation, becoming lost, or becoming academically dropped.

For example, it can be shown that, according to this model, 15.21% of students entering in CI 1 will go on to graduate from this campus, given enough time. The same group of students have a 70.03% chance of leaving voluntarily without graduating, and a 14.57% chance of being academically dropped. Newly enrolled students in Classification Indexes of 6 or greater have a 50% or greater chance of graduating. Students newly enrolled in CI 8 have a 92.62% change of graduating.

Next, the distribution of incoming students from the initial state matrix is applied to the steady-state partition in order to obtain an overall graduation rate, over time. For example, 11.11% of incoming students are First Time Freshmen; 12.16% of First Time Freshmen are expected to graduate; therefore, 1.35% of graduating students began at this campus as First Time Freshmen.

**Table 10.03  
Steady-State Partition Applied to Initial Distribution**

		ND	FF	1	2	3	4	5	6	7	8	
Incoming		0.0975	0.1111	0.6214	0.0474	0.0282	0.0411	0.0130	0.0081	0.0054	0.0268	
X	D	0.0896	0.1781	0.1457	0.0787	0.0520	0.0340	0.0311	0.0268	0.0122	0.0104	
X	GR	0.0402	0.1216	0.1521	0.2555	0.3514	0.4928	0.4339	0.5441	0.6870	0.9262	
X	L	0.8703	0.7003	0.7022	0.6657	0.5966	0.4732	0.5350	0.4291	0.3008	0.0634	TOTAL
Of	D	0.0087	0.0198	0.0906	0.0037	0.0015	0.0014	0.0004	0.0002	0.0001	0.0003	0.1266
Total	GR	0.0039	0.0135	0.0945	0.0121	0.0099	0.0203	0.0056	0.0044	0.0037	0.0248	0.1928
	L	0.0848	0.0778	0.4363	0.0315	0.0168	0.0195	0.0069	0.0035	0.0016	0.0017	0.6805
											check	1.0000

Over enough time, 19.28% of the distribution of incoming students will graduate, 68.05% will leave the university voluntarily without graduating, and 12.66% will be academically dropped.

Variations of this same steady-state model were developed to show the eventual outcomes of other specific groups of students, particularly students grouped by age, full- or part-time status, and degree group. The next several sections will detail the development of these models, and analyze the results.

### **10.1 The Steady-State Model Applied to Hours Enrolled and Classification Index**

Using the same methods to develop the steady-state model above, data was sorted and summarized to create a transition matrix whose states were first grouped by full-time or part-time enrollment, and then by Classification Index within those two groupings. The partitioned steady state matrix, by hours enrolled and Classification Index follows. The matrices required to develop the steady-state matrix for this grouping of data will also follow.

**Table 10.1.1  
Partitioned Transition Matrix by Hours Registered and CI**

		CURRENT																							
		Full-time												Part-time											
		X D	X GR	X L	ND	FF	1	2	3	4	5	6	7	8	ND	FF	1	2	3	4	5	6	7	8	
NEXT	Full-time	X D	1.0000		0.0806	0.1538	0.0705	0.0276	0.0169	0.0105	0.0031	0.0049		0.0018	0.0848	0.2847	0.1051	0.0299	0.0165	0.0100	0.0084	0.0164	0.0030	0.0013	
		X GR		1.0000										0.0077	0.5662				0.0011	0.1594	0.0209	0.0117	0.0122	0.4116	
		X L			1.0000	0.2419	0.1381	0.1363	0.1472	0.1408	0.0929	0.1011	0.1024	0.1673	0.0236	0.4214	0.2918	0.2334	0.2249	0.2196	0.1253	0.2469	0.2061	0.1738	0.0264
Full-time	ND				0.1452																				
	FF													0.0152											
	1				0.1774	0.4063	0.2802	0.0086	0.0045	0.0018	0.0047	0.0049	0.0038	0.0054	0.0455	0.1459	0.0943	0.0023		0.0007		0.0047	0.0061	0.0040	
	2				0.0323	0.1893	0.3204	0.0988	0.0056	0.0023	0.0093	0.0407	0.0019	0.0054			0.0501	0.0575	0.0023	0.0003	0.0021	0.0047	-	-	
	3						0.0227	0.5032	0.1025	0.0092	0.0016	0.0033	0.0038	0.0018			0.0023	0.0715	0.0616	0.0010	0.0042	-	0.0030	-	
	4						0.0036	0.0492	0.5163	0.3407	0.0202	0.0179	0.0096	0.0073			0.0007	0.0027	0.0696	0.0675	0.0063	0.0047	0.0061	0.0040	
	5						0.0007	0.0039	0.0242	0.1983	0.1244	0.0065	0.0038	-			-	-	0.0029	0.0152	0.0649	0.0047	-	-	
	6						0.0011	0.0035	0.0051	0.0531	0.5163	0.1073	0.0058	0.0018			0.0009	0.0005	0.0029	0.0031	0.0649	0.0796	-	0.0026	
	7				0.0161		0.0002	0.0004	0.0006	0.0279	0.0747	0.5187	0.0788	0.0018					0.0005		0.0010	0.0021	0.0726	0.0640	0.0013
	8								0.0006	0.0179	0.0140	0.0780	0.5212	0.2323					0.0005		0.0021	0.0021	0.0047	0.0915	0.1240
	Part-time	ND				0.1613		0.0007	0.0004							0.3500		0.0002							
FF														0.0063											
1					0.1452	0.0947	0.0925	0.0030	0.0006	0.0014	0.0016				0.0643	0.2740	0.3391	0.0095	0.0034	0.0041	0.0105	0.0047	0.0030	0.0106	
2						0.0178	0.0672	0.0393	0.0006	0.0005	0.0031				0.0027	0.0036	0.1617	0.3303	0.0120	0.0031	0.0126	0.0047	-	0.0026	
3							0.0031	0.1036	0.0462	0.0023	-	0.0033	0.0019		0.0009		0.0063	0.2529	0.3611	0.0100	0.0042	0.0117	0.0122	0.0040	
4							0.0009	0.0099	0.1295	0.1218	0.0093	0.0065	-	0.0036	0.0009		0.0041	0.0136	0.2447	0.5072	0.0335	0.0258	0.0335	0.0567	
5								0.0009	0.0045	0.0398	0.0280	-	-	0.0018	0.0009		0.0014	0.0009	0.0040	0.0530	0.2866	0.0117	0.0122	0.0026	
6								-	0.0011	0.0078	0.0731	0.0244	0.0038	-	-		0.0005	0.0023	0.0006	0.0169	0.2113	0.2974	0.0091	0.0053	
7								0.0006	0.0032	0.0078	0.0650	0.0288	-	-		0.0002	0.0005	0.0006	0.0079	0.0167	0.2201	0.3567	0.0092		
8							0.0004	0.0006	0.0041	0.0016	0.0146	0.1615	0.1470	0.0009					0.0117	0.0021	0.0141	0.2134	0.3338		
check		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	

**Table 10.1.2**  
**Steady-State Calculation by Hours Registered and CI**

**Identity Matrix**

1.0000																				
	1.0000																			
		1.0000																		
			1.0000																	
				1.0000																
					1.0000															
						1.0000														
							1.0000													
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															1.0000					
																1.0000				
																	1.0000			
																		1.0000		
																			1.0000	
																				1.0000

**Partition R of Transition Matrix**

0.1452											0.0152									
											0.0063									
0.1774	0.4063	0.2802	0.0086	0.0045	0.0018	0.0047	0.0049	0.0038	0.0054	0.0455	0.1459	0.0943	0.0023		0.0007	0.0047	0.0061	0.0040		
0.0323	0.1893	0.3204	0.0988	0.0056	0.0023	0.0093	0.0407	0.0019	0.0054			0.0501	0.0575	0.0023	0.0003	0.0021	0.0047	-	-	
		0.0227	0.5032	0.1025	0.0092	0.0016	0.0033	0.0038	0.0018			0.0023	0.0715	0.0616	0.0010	0.0042	-	0.0030	-	
		0.0036	0.0492	0.5163	0.3407	0.0202	0.0179	0.0096	0.0073			0.0007	0.0027	0.0696	0.0675	0.0063	0.0047	0.0061	0.0040	
		0.0007	0.0039	0.0242	0.1983	0.1244	0.0065	0.0038	-			-	-	0.0029	0.0152	0.0649	0.0047	-	-	
		0.0011	0.0035	0.0051	0.0531	0.5163	0.1073	0.0058	0.0018			0.0009	0.0005		0.0031	0.0649	0.0796	-	0.0026	
0.0161		0.0002	0.0004	0.0006	0.0279	0.0747	0.5187	0.0788	0.0018				0.0005		0.0010	0.0021	0.0726	0.0640	0.0013	
					0.0179	0.0140	0.0780	0.5212	0.2323				0.0005		0.0021	0.0021	0.0047	0.0915	0.1240	
0.1613		0.0007	0.0004			0.0016				0.3500		0.0002			0.0003					
										0.0063										
0.1452	0.0947	0.0925	0.0030	0.0006	0.0014	0.0016				0.0643	0.2740	0.3391	0.0095	0.0034	0.0041	0.0105	0.0047	0.0030	0.0106	
	0.0178	0.0672	0.0393	0.0006	0.0005	0.0031				0.0027	0.0036	0.1617	0.3303	0.0120	0.0031	0.0126	0.0047	-	0.0026	
		0.0031	0.1036	0.0462	0.0023	-	0.0033	0.0019		0.0009		0.0063	0.2529	0.3611	0.0100	0.0042	0.0117	0.0122	0.0040	
		0.0009	0.0099	0.1295	0.1218	0.0093	0.0065	-	0.0036	0.0009		0.0041	0.0136	0.2447	0.5072	0.0335	0.0258	0.0335	0.0567	
			0.0009	0.0045	0.0398	0.0280	-	-	0.0018	0.0009		0.0014	0.0009	0.0040	0.0530	0.2866	0.0117	0.0122	0.0026	
			-	0.0011	0.0078	0.0731	0.0244	0.0038	-	-		0.0005	0.0023	0.0006	0.0169	0.2113	0.2974	0.0091	0.0053	
			-	0.0006	0.0032	0.0078	0.0650	0.0288	-	-		0.0002	0.0005	0.0006	0.0079	0.0167	0.2201	0.3567	0.0092	
			0.0004	0.0006	0.0041	0.0016	0.0146	0.1615	0.1470	0.0009					0.0117	0.0021	0.0141	0.2134	0.3338	



**Table 10.1.2 (continued)**  
**(I – R) Calculation**

0.855	-	-	-	-	-	-	-	-	-	-	(0.015)	-	-	-	-	-	-	-	-	-
-	1.000	-	-	-	-	-	-	-	-	-	(0.006)	-	-	-	-	-	-	-	-	-
(0.177)	(0.406)	0.720	(0.009)	(0.005)	(0.002)	(0.005)	(0.005)	(0.004)	(0.005)	(0.046)	(0.146)	(0.094)	(0.002)	-	(0.001)	-	(0.005)	(0.006)	(0.004)	
(0.032)	(0.189)	(0.320)	0.901	(0.006)	(0.002)	(0.009)	(0.041)	(0.002)	(0.005)	-	-	(0.050)	(0.057)	(0.002)	(0.000)	(0.002)	(0.005)	-	-	
-	-	(0.023)	(0.503)	0.898	(0.009)	(0.002)	(0.003)	(0.004)	(0.002)	-	-	(0.002)	(0.071)	(0.062)	(0.001)	(0.004)	-	(0.003)	-	
-	-	(0.004)	(0.049)	(0.516)	0.659	(0.020)	(0.018)	(0.010)	(0.007)	-	-	(0.001)	(0.003)	(0.070)	(0.067)	(0.006)	(0.005)	(0.006)	(0.004)	
-	-	(0.001)	(0.004)	(0.024)	(0.198)	0.876	(0.007)	(0.004)	-	-	-	-	-	(0.003)	(0.015)	(0.065)	(0.005)	-	-	
-	-	(0.001)	(0.003)	(0.005)	(0.053)	(0.516)	0.893	(0.006)	(0.002)	-	-	(0.001)	(0.000)	-	(0.003)	(0.065)	(0.080)	-	(0.003)	
(0.016)	-	(0.000)	(0.000)	(0.001)	(0.028)	(0.075)	(0.519)	0.921	(0.002)	-	-	-	(0.000)	-	(0.001)	(0.002)	(0.073)	(0.064)	(0.001)	
-	-	-	-	-	(0.018)	(0.014)	(0.078)	(0.521)	0.768	-	-	-	(0.000)	-	(0.002)	(0.002)	(0.075)	(0.091)	(0.124)	
(0.161)	-	(0.001)	(0.000)	-	-	(0.002)	-	-	-	0.650	-	(0.000)	-	-	(0.000)	-	-	-	-	
-	-	-	-	-	-	-	-	-	-	(0.006)	1.000	-	-	-	-	-	-	-	-	
(0.145)	(0.095)	(0.093)	(0.003)	(0.001)	(0.001)	(0.002)	-	-	-	(0.064)	(0.274)	0.861	(0.010)	(0.003)	(0.004)	(0.010)	(0.005)	(0.003)	(0.011)	
-	(0.018)	(0.067)	(0.039)	(0.001)	(0.000)	(0.003)	-	-	-	(0.003)	(0.004)	(0.162)	0.670	(0.012)	(0.003)	(0.013)	(0.005)	-	(0.003)	
-	-	(0.003)	(0.104)	(0.046)	(0.002)	-	(0.003)	(0.002)	-	(0.001)	-	(0.006)	(0.253)	0.639	(0.010)	(0.004)	(0.012)	(0.012)	(0.004)	
-	-	(0.001)	(0.010)	(0.130)	(0.122)	(0.009)	(0.007)	-	(0.004)	(0.001)	-	(0.004)	(0.014)	(0.245)	0.493	(0.033)	(0.026)	(0.034)	(0.057)	
-	-	-	(0.001)	(0.005)	(0.040)	(0.028)	-	-	(0.002)	(0.001)	-	(0.001)	(0.001)	(0.004)	(0.053)	0.713	(0.012)	(0.012)	(0.003)	
-	-	-	-	(0.001)	(0.008)	(0.073)	(0.024)	(0.004)	-	-	-	(0.000)	(0.002)	(0.001)	(0.017)	(0.211)	0.703	(0.009)	(0.005)	
-	-	-	-	(0.001)	(0.003)	(0.008)	(0.065)	(0.029)	-	-	-	(0.000)	(0.000)	(0.001)	(0.008)	(0.017)	(0.220)	0.643	(0.009)	
-	-	-	(0.000)	(0.001)	(0.004)	(0.002)	(0.015)	(0.162)	(0.147)	(0.001)	-	-	-	-	(0.012)	(0.002)	(0.014)	(0.213)	0.666	

**(I – R)<sup>-1</sup> Inverse of (I – R) Matrix**

1.175	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.027	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.002	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.361	0.608	1.433	0.031	0.023	0.020	0.029	0.025	0.018	0.014	0.138	0.267	0.211	0.017	0.011	0.012	0.018	0.026	0.026	0.016	0.016
0.203	0.461	0.548	1.148	0.038	0.038	0.064	0.071	0.017	0.015	0.069	0.134	0.194	0.116	0.022	0.017	0.032	0.031	0.017	0.012	0.012
0.145	0.306	0.385	0.682	1.161	0.049	0.052	0.055	0.020	0.014	0.052	0.104	0.170	0.238	0.135	0.024	0.037	0.030	0.023	0.013	0.013
0.164	0.336	0.425	0.728	1.019	1.650	0.144	0.124	0.060	0.038	0.061	0.120	0.209	0.336	0.390	0.263	0.099	0.087	0.078	0.051	0.051
0.048	0.098	0.124	0.210	0.288	0.400	1.194	0.047	0.023	0.011	0.018	0.035	0.062	0.101	0.124	0.114	0.140	0.039	0.028	0.019	0.019
0.045	0.090	0.115	0.192	0.258	0.358	0.728	1.171	0.031	0.015	0.017	0.033	0.059	0.095	0.116	0.116	0.237	0.168	0.035	0.025	0.025
0.059	0.076	0.097	0.161	0.218	0.305	0.541	0.687	1.116	0.015	0.015	0.028	0.050	0.082	0.100	0.103	0.206	0.255	0.142	0.025	0.025
0.058	0.086	0.109	0.181	0.244	0.342	0.545	0.677	0.849	1.368	0.017	0.032	0.057	0.095	0.118	0.129	0.238	0.323	0.397	0.284	0.284
0.292	0.001	0.002	0.002	0.001	0.001	0.003	0.000	0.000	0.000	1.546	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000
0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.344	0.239	0.214	0.023	0.018	0.019	0.018	0.014	0.012	0.008	0.184	0.457	1.555	0.036	0.022	0.023	0.036	0.026	0.025	0.031	0.031
0.136	0.179	0.236	0.089	0.021	0.020	0.022	0.014	0.009	0.006	0.070	0.154	0.415	1.528	0.045	0.025	0.046	0.026	0.016	0.018	0.018
0.108	0.182	0.230	0.288	0.121	0.042	0.042	0.040	0.018	0.010	0.049	0.099	0.231	0.655	1.618	0.057	0.056	0.063	0.052	0.026	0.026
0.155	0.290	0.366	0.567	0.670	0.509	0.175	0.153	0.100	0.067	0.065	0.125	0.253	0.539	0.969	2.176	0.234	0.225	0.245	0.222	0.222
0.026	0.050	0.063	0.102	0.131	0.152	0.078	0.028	0.017	0.013	0.012	0.021	0.040	0.072	0.112	0.186	1.444	0.059	0.056	0.028	0.028
0.022	0.042	0.053	0.085	0.111	0.135	0.187	0.068	0.022	0.010	0.009	0.017	0.033	0.060	0.082	0.130	0.470	1.469	0.054	0.030	0.030
0.020	0.036	0.046	0.074	0.098	0.123	0.186	0.182	0.070	0.012	0.008	0.014	0.027	0.048	0.066	0.097	0.240	0.541	1.597	0.041	0.041
0.039	0.060	0.076	0.126	0.166	0.220	0.338	0.405	0.484	0.311	0.014	0.023	0.042	0.072	0.096	0.131	0.204	0.346	0.641	1.588	1.588

**Table 10.1.2 (continued)  
S Partition of Transition Matrix**

0.0806	0.1538	0.0705	0.0276	0.0169	0.0105	0.0031	0.0049	-	0.0018	0.0848	0.2847	0.1051	0.0299	0.0165	0.0100	0.0084	0.0164	0.0030	0.0013
-	-	-	-	-	0.0646	0.0047	0.0016	0.0077	0.5662	-	-	-	-	0.0011	0.1594	0.0209	0.0117	0.0122	0.4116
0.2419	0.1381	0.1363	0.1472	0.1408	0.0929	0.1011	0.1024	0.1673	0.0236	0.4214	0.2918	0.2334	0.2249	0.2196	0.1253	0.2469	0.2061	0.1738	0.0264

**$S(I - R)^{-1}$  Calculation**

0.2003	0.2569	0.1672	0.0733	0.0504	0.0369	0.0245	0.0196	0.0087	0.0072	0.1741	0.3667	0.2092	0.0804	0.0517	0.0358	0.0351	0.0391	0.018	0.0123
0.0865	0.1444	0.1829	0.2985	0.3883	0.4832	0.5021	0.5939	0.7102	0.9163	0.0308	0.0565	0.1062	0.1957	0.2933	0.499	0.3037	0.3945	0.5551	0.8548
0.7132	0.5987	0.6499	0.6282	0.5613	0.4799	0.4734	0.3866	0.2811	0.0764	0.7952	0.5768	0.6847	0.7239	0.655	0.4652	0.6612	0.5664	0.4269	0.1329

**Final Steady-State Partition**

		Full-time								Part-time											
		ND	FF	1	2	3	4	5	6	7	8	ND	FF	1	2	3	4	5	6	7	8
X	D	0.2003	0.2569	0.1672	0.0733	0.0504	0.0369	0.0245	0.0196	0.0087	0.0072	0.1741	0.3667	0.2092	0.0804	0.0517	0.0358	0.0351	0.0391	0.018	0.0123
X	GR	0.0865	0.1444	0.1829	0.2985	0.3883	0.4832	0.5021	0.5939	0.7102	0.9163	0.0308	0.0565	0.1062	0.1957	0.2933	0.499	0.3037	0.3945	0.5551	0.8548
X	L	0.7132	0.5987	0.6499	0.6282	0.5613	0.4799	0.4734	0.3866	0.2811	0.0764	0.7952	0.5768	0.6847	0.7239	0.655	0.4652	0.6612	0.5664	0.4269	0.1329
check		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The distribution of incoming students grouped by hours registered and CI is applied to the steady-state partition in order to obtain an overall graduation rate, over time.

Over time, the graduation rates for full-time and part-time students vary little. Based on this model, full-time students can be expected to graduate at a rate of 20.01% while part-time students have an expected graduation rate of 24.29%. Of full-time students, 62.91% can be expected to leave the university without graduating, and 17.08% will be academically dropped. Of part-time students, 60.83% will leave the university voluntarily without graduating, and 14.88% will be academically dropped.

**Table 10.1.3  
Steady-State Partition Applied to Initial Distribution**

		Full-time										
		ND	FF	1	2	3	4	5	6	7	8	
Incoming		0.0097	0.1604	0.7376	0.0328	0.0162	0.0187	0.0101	0.0058	0.0022	0.0065	
X	D	0.2003	0.2569	0.1672	0.0733	0.0504	0.0369	0.0245	0.0196	0.0087	0.0072	
X	GR	0.0865	0.1444	0.1829	0.2985	0.3883	0.4832	0.5021	0.5939	0.7102	0.9163	
X	L	0.7132	0.5987	0.6499	0.6282	0.5613	0.4799	0.4734	0.3866	0.2811	0.0764	<b>TOTAL</b>
Of	D	0.0019	0.0412	0.1233	0.0024	0.0008	0.0007	0.0002	0.0001	0.0000	0.0000	0.1708
Total	GR	0.0008	0.0232	0.1349	0.0098	0.0063	0.0091	0.0051	0.0034	0.0015	0.0059	0.2001
	L	0.0069	0.0960	0.4793	0.0206	0.0091	0.0090	0.0048	0.0022	0.0006	0.0005	0.6291

1.0000

		Part-time										
		ND	FF	1	2	3	4	5	6	7	8	
Incoming		0.1746	0.0678	0.5193	0.0602	0.0387	0.0608	0.0155	0.0101	0.0082	0.0447	
X	D	0.1741	0.3667	0.2092	0.0804	0.0517	0.0358	0.0351	0.0391	0.0180	0.0123	
X	GR	0.0308	0.0565	0.1062	0.1957	0.2933	0.4990	0.3037	0.3945	0.5551	0.8548	
X	L	0.7952	0.5768	0.6847	0.7239	0.6550	0.4652	0.6612	0.5664	0.4269	0.1329	<b>TOTAL</b>
Of	D	0.0350	0.0174	0.0868	0.0044	0.0019	0.0022	0.0004	0.0002	0.0001	0.0003	0.1488
Total	GR	0.0151	0.0098	0.0950	0.0180	0.0150	0.0294	0.0078	0.0060	0.0059	0.0409	0.2429
	L	0.1245	0.0406	0.3375	0.0378	0.0217	0.0292	0.0074	0.0039	0.0023	0.0034	0.6083

1.0000

## 10.2 The Steady-State Model Applied to Age and Classification Index

The transition matrix is regrouped, first by Age (Traditional and Non-traditional), and then by Classification Index within those two groupings. The partitioned steady state matrix, by age and Classification Index follows. The matrices required to develop the steady-state matrix for this grouping of data will also follow.

It should be noticed that, over time, as the steady-state model represents, all students will become non-traditionally aged, or age beyond 25 years. Further, students who are initially non-traditionally aged cannot become traditionally aged. This impossible movement is reflected in the transition matrix with the block of 0.0000 probabilities for students non-traditionally aged in the Current state.

**Table 10.2.1  
Partitioned Transition Matrix by Age and CI**

		CURRENT																						
		Traditional												Non-Traditional										
		X D	X GR	X L	ND	FF	1	2	3	4	5	6	7	8	ND	FF	1	2	3	4	5	6	7	8
X	D	1.0000			0.0886	0.2073	0.1018	0.0362	0.0253	0.0170	0.0078	0.0158		0.0020	0.0777	0.1375	0.0496	0.0150	0.0042	0.0028	0.0042	0.0021	0.0025	0.0013
	GR		1.0000		-	-	-	-	-	0.1026	0.0155	0.0053	0.0110	0.5930	-	-	-	-	0.0014	0.1309	0.0063	0.0042	0.0076	0.4356
	L			1.0000	0.3892	0.1819	0.1801	0.1827	0.1780	0.0954	0.1366	0.1090	0.1744		0.4329	0.2875	0.2171	0.1900	0.1827	0.1329	0.1987	0.1878	0.1641	
Traditional	ND				0.3544		0.0005	0.0003		0.0004	0.0016													
	FF				0.0174		-	0.0003		-	-													
	1				0.1345	0.4683	0.3840	0.0126	0.0029	0.0034	0.0109	0.0018	0.0044	0.0039										
	2				0.0032	0.1368	0.2933	0.2107	0.0095	0.0030	0.0109	0.0105	-	0.0020										
	3				0.0016		0.0160	0.4858	0.2305	0.0121	0.0031	0.0070	0.0088	-										
	4						0.0039	0.0393	0.5079	0.4818	0.0233	0.0246	0.0155	0.0117										
	5						0.0006	0.0020	0.0177	0.1730	0.1894	0.0123	0.0066	0.0020										
	6						0.0011	0.0020	0.0043	0.0397	0.5000	0.1687	0.0110	0.0059										
Non-Traditional	7						0.0002	0.0007		0.0208	0.0512	0.5255	0.1435	0.0020										
	8						0.0002	0.0003		0.0144	0.0140	0.0826	0.5894	0.3405										
	ND				0.0079										0.3675		0.0008			0.0004				
	FF				-										0.0053		-			-				
	1				0.0032	0.0056	0.0113		0.0005						0.1025	0.4625	0.4106	0.0100	0.0063	0.0049	0.0042	0.0127	0.0076	0.0152
	2						0.0064	0.0079	-						0.0071	0.1125	0.2935	0.3364	0.0112	0.0032	0.0167	0.0084	0.0025	0.0051
	3						0.0006	0.0167	0.0100	0.0008				0.0022	0.0018		0.0182	0.4056	0.3487	0.0089	0.0063	0.0127	0.0076	0.0051
	4						-	0.0017	0.0129	0.0280	0.0031	0.0053	-	0.0039	0.0018		0.0066	0.0336	0.4233	0.5466	0.0439	0.0232	0.0278	0.0568
5						-	0.0003	0.0005	0.0053	0.0109	0.0018	-	-	0.0018		0.0019	0.0037	0.0174	0.1005	0.2866	0.0105	0.0076	0.0025	
6						0.0002	0.0003		0.0004	0.0140	0.0105	-	-	0.0018		0.0012	0.0044	0.0021	0.0344	0.3766	0.3017	0.0076	0.0051	
7									0.0011	0.0078	0.0176	0.0088	-			0.0004	0.0006	0.0021	0.0150	0.0502	0.4008	0.3157	0.0101	
8									0.0008		0.0018	0.0243	0.0333				0.0006	0.0007	0.0194	0.0063	0.0359	0.4495	0.4634	
check		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

**Table 10.2.2**  
**Steady-State Calculation by State and CI**

**Identity Matrix**

1.0000																				
	1.0000																			
		1.0000																		
			1.0000																	
				1.0000																
					1.0000															
						1.0000														
							1.0000													
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															1.0000					
																1.0000				
																	1.0000			
																		1.0000		
																			1.0000	
																				1.0000

**Partition R of Transition Matrix**

0.3544		0.0005	0.0003		0.0004	0.0016														
0.0174		-	0.0003		-	-														
0.1345	0.4683	0.3840	0.0126	0.0029	0.0034	0.0109	0.0018	0.0044	0.0039											
0.0032	0.1368	0.2933	0.2107	0.0095	0.0030	0.0109	0.0105	-	0.0020											
0.0016		0.0160	0.4858	0.2305	0.0121	0.0031	0.0070	0.0088	-											
		0.0039	0.0393	0.5079	0.4818	0.0233	0.0246	0.0155	0.0117											
		0.0006	0.0020	0.0177	0.1730	0.1894	0.0123	0.0066	0.0020											
		0.0011	0.0020	0.0043	0.0397	0.5000	0.1687	0.0110	0.0059											
		0.0002	0.0007		0.0208	0.0512	0.5255	0.1435	0.0020											
		0.0002	0.0003		0.0144	0.0140	0.0826	0.5894	0.3405											
0.0079										0.3675		0.0008			0.0004					
-										0.0053		-			-					
0.0032	0.0056	0.0113		0.0005						0.1025	0.4625	0.4106	0.0100	0.0063	0.0049	0.0042	0.0127	0.0076	0.0152	
		0.0064	0.0079	-						0.0071	0.1125	0.2935	0.3364	0.0112	0.0032	0.0167	0.0084	0.0025	0.0051	
		0.0006	0.0167	0.0100	0.0008			0.0022		0.0018		0.0182	0.4056	0.3487	0.0089	0.0063	0.0127	0.0076	0.0051	
		-	0.0017	0.0129	0.0280	0.0031	0.0053	-	0.0039	0.0018		0.0066	0.0336	0.4233	0.5466	0.0439	0.0232	0.0278	0.0568	
		0.0002	0.0003	0.0005	0.0053	0.0109	0.0018	-	-	0.0018		0.0019	0.0037	0.0174	0.1005	0.2866	0.0105	0.0076	0.0025	
					0.0004	0.0140	0.0105	-	-	0.0018		0.0012	0.0044	0.0021	0.0344	0.3766	0.3017	0.0076	0.0051	
					0.0011	0.0078	0.0176	0.0088	-			0.0004	0.0006	0.0021	0.0150	0.0502	0.4008	0.3157	0.0101	
					0.0008		0.0018	0.0243	0.0333				0.0006	0.0007	0.0194	0.0063	0.0359	0.4495	0.4634	

**Table 10.2.2 (continued)**  
**(I – R) Calculation**

0.646	-	(0.000)	(0.000)	-	(0.000)	(0.002)	-	-	-	-	-	-	-	-	-	-	-	-	-
(0.017)	1.000	-	(0.000)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(0.134)	(0.468)	0.616	(0.013)	(0.003)	(0.003)	(0.011)	(0.002)	(0.004)	(0.004)	-	-	-	-	-	-	-	-	-	-
(0.003)	(0.137)	(0.293)	0.789	(0.010)	(0.003)	(0.011)	(0.011)	-	(0.002)	-	-	-	-	-	-	-	-	-	-
(0.002)	-	(0.016)	(0.486)	0.769	(0.012)	(0.003)	(0.007)	(0.009)	-	-	-	-	-	-	-	-	-	-	-
-	-	(0.004)	(0.039)	(0.508)	0.518	(0.023)	(0.025)	(0.015)	(0.012)	-	-	-	-	-	-	-	-	-	-
-	-	(0.001)	(0.002)	(0.018)	(0.173)	0.811	(0.012)	(0.007)	(0.002)	-	-	-	-	-	-	-	-	-	-
-	-	(0.001)	(0.002)	(0.004)	(0.040)	(0.500)	0.831	(0.011)	(0.006)	-	-	-	-	-	-	-	-	-	-
-	-	(0.000)	(0.001)	-	(0.021)	(0.051)	(0.525)	0.857	(0.002)	-	-	-	-	-	-	-	-	-	-
-	-	(0.000)	(0.000)	-	(0.014)	(0.014)	(0.083)	(0.589)	0.659	-	-	-	-	-	-	-	-	-	-
(0.008)	-	-	-	-	-	-	-	-	-	0.633	-	(0.001)	-	-	(0.000)	-	-	-	-
-	-	-	-	-	-	-	-	-	-	(0.005)	1.000	-	-	-	-	-	-	-	-
(0.003)	(0.006)	(0.011)	-	(0.000)	-	-	-	-	-	(0.102)	(0.463)	0.589	(0.010)	(0.006)	(0.005)	(0.004)	(0.013)	(0.008)	(0.015)
-	-	(0.006)	(0.008)	-	-	-	-	-	-	(0.007)	(0.113)	(0.294)	0.664	(0.011)	(0.003)	(0.017)	(0.008)	(0.003)	(0.005)
-	-	(0.001)	(0.017)	(0.010)	(0.001)	-	-	(0.002)	-	(0.002)	-	(0.018)	(0.406)	0.651	(0.009)	(0.006)	(0.013)	(0.008)	(0.005)
-	-	-	(0.002)	(0.013)	(0.028)	(0.003)	(0.005)	-	(0.004)	(0.002)	-	(0.007)	(0.034)	(0.423)	0.453	(0.044)	(0.023)	(0.028)	(0.057)
-	-	-	(0.000)	(0.000)	(0.005)	(0.011)	(0.002)	-	-	(0.002)	-	(0.002)	(0.004)	(0.017)	(0.100)	0.713	(0.011)	(0.008)	(0.003)
-	-	(0.000)	(0.000)	-	(0.000)	(0.014)	(0.011)	-	-	(0.002)	-	(0.001)	(0.004)	(0.002)	(0.034)	(0.377)	0.698	(0.008)	(0.005)
-	-	-	-	-	(0.001)	(0.008)	(0.018)	(0.009)	-	-	-	(0.000)	(0.001)	(0.002)	(0.015)	(0.050)	(0.401)	0.684	(0.010)
-	-	-	-	-	(0.001)	-	(0.002)	(0.024)	(0.033)	-	-	-	(0.001)	(0.001)	(0.019)	(0.006)	(0.036)	(0.449)	0.537

**(I – R)<sup>-1</sup> Inverse of (I – R) Matrix**

1.550	0.001	0.002	0.002	0.002	0.002	0.003	0.000	0.000	0.000	-	-	-	-	-	-	-	-	-	-
0.027	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-	-	-	-	-	-	-	-	-	-
0.364	0.776	1.645	0.044	0.025	0.026	0.036	0.017	0.017	0.011	-	-	-	-	-	-	-	-	-	-
0.149	0.470	0.622	1.303	0.040	0.030	0.045	0.027	0.011	0.009	-	-	-	-	-	-	-	-	-	-
0.107	0.320	0.437	0.843	1.353	0.061	0.051	0.041	0.023	0.007	-	-	-	-	-	-	-	-	-	-
0.123	0.366	0.504	0.954	1.370	2.047	0.181	0.148	0.088	0.044	-	-	-	-	-	-	-	-	-	-
0.030	0.089	0.122	0.230	0.329	0.447	1.292	0.060	0.032	0.014	-	-	-	-	-	-	-	-	-	-
0.026	0.076	0.105	0.195	0.275	0.374	0.798	1.263	0.047	0.022	-	-	-	-	-	-	-	-	-	-
0.021	0.062	0.085	0.158	0.222	0.307	0.573	0.784	1.203	0.019	-	-	-	-	-	-	-	-	-	-
0.025	0.075	0.104	0.192	0.270	0.375	0.643	0.863	1.084	1.537	-	-	-	-	-	-	-	-	-	-
0.019	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.581	0.001	0.003	0.001	0.001	0.002	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.020	0.028	0.036	0.008	0.008	0.007	0.008	0.007	0.005	0.004	0.288	0.806	1.728	0.058	0.048	0.043	0.058	0.073	0.060	0.056
0.015	0.027	0.042	0.023	0.008	0.008	0.008	0.007	0.005	0.003	0.149	0.538	0.782	1.564	0.068	0.048	0.084	0.067	0.048	0.044
0.017	0.038	0.055	0.067	0.035	0.016	0.015	0.012	0.010	0.004	0.108	0.366	0.549	0.999	1.612	0.079	0.102	0.096	0.067	0.051
0.032	0.081	0.114	0.175	0.184	0.179	0.082	0.072	0.048	0.035	0.131	0.417	0.631	1.109	1.592	2.391	0.401	0.348	0.336	0.308
0.007	0.018	0.025	0.040	0.045	0.050	0.037	0.018	0.009	0.006	0.027	0.074	0.114	0.194	0.271	0.348	1.482	0.087	0.072	0.054
0.007	0.018	0.025	0.040	0.046	0.053	0.064	0.035	0.010	0.007	0.027	0.068	0.104	0.175	0.234	0.311	0.831	1.513	0.082	0.059
0.007	0.018	0.026	0.043	0.052	0.063	0.087	0.069	0.027	0.008	0.022	0.057	0.088	0.148	0.200	0.266	0.614	0.914	1.542	0.074
0.010	0.028	0.039	0.067	0.083	0.104	0.150	0.157	0.147	0.104	0.026	0.070	0.106	0.181	0.246	0.334	0.602	0.880	1.310	1.942

**Table 10.2.2 (continued)  
S Partition of Transition Matrix**

0.0886	0.2073	0.1018	0.0362	0.0253	0.0170	0.0078	0.0158	-	0.0020	0.0777	0.1375	0.0496	0.0150	0.0042	0.0028	0.0042	0.0021	0.0025	0.0013
-	-	-	-	-	0.1026	0.0155	0.0053	0.0110	0.5930	-	-	-	-	0.0014	0.1309	0.0063	0.0042	0.0076	0.4356
0.3892	0.1819	0.1801	0.1827	0.1780	0.0954	0.1366	0.1090	0.1744	-	0.4329	0.2875	0.2171	0.1900	0.1827	0.1329	0.1987	0.1878	0.1641	-

**$S(I - R)^{-1}$  Calculation**

0.1938	0.3222	0.2159	0.0967	0.0708	0.0522	0.0355	0.0297	0.0082	0.0063	0.1417	0.189	0.1028	0.0354	0.0172	0.0133	0.016	0.013	0.011	0.0076
0.0372	0.1077	0.1492	0.2709	0.3711	0.5148	0.5078	0.622	0.7364	0.9667	0.0289	0.0865	0.1315	0.2287	0.3222	0.4641	0.3322	0.4431	0.6272	0.8873
0.769	0.5701	0.6349	0.6324	0.558	0.433	0.4567	0.3484	0.2554	0.027	0.8294	0.7244	0.7657	0.7359	0.6606	0.5226	0.6519	0.5439	0.3618	0.1051

**Final Steady-State Partition**

		Traditional										Non-Traditional									
		ND	FF	1	2	3	4	5	6	7	8	ND	FF	1	2	3	4	5	6	7	8
X	D	0.1938	0.3222	0.2159	0.0967	0.0708	0.0522	0.0355	0.0297	0.0082	0.0063	0.1417	0.189	0.1028	0.0354	0.0172	0.0133	0.016	0.013	0.011	0.0076
X	GR	0.0372	0.1077	0.1492	0.2709	0.3711	0.5148	0.5078	0.622	0.7364	0.9667	0.0289	0.0865	0.1315	0.2287	0.3222	0.4641	0.3322	0.4431	0.6272	0.8873
X	L	0.769	0.5701	0.6349	0.6324	0.558	0.433	0.4567	0.3484	0.2554	0.027	0.8294	0.7244	0.7657	0.7359	0.6606	0.5226	0.6519	0.5439	0.3618	0.1051
check		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The distribution of incoming students grouped by age and CI is applied to the steady-state partition in order to obtain an overall graduation rate, over time.

Over time, the graduation rates for traditionally and non-traditionally aged students vary in favor of the non-traditionally aged students. Based on this model, traditionally aged students can be expected to graduate at a rate of 16.37% while non-traditionally aged students have an expected graduation rate of 25.63%. Of traditionally aged students, 62.11% can be expected to leave the university without graduating, and 21.52% will be academically dropped. Of non-traditionally aged students, 57.89% will leave the university voluntarily without graduating, and 16.48% will be academically dropped.

**Table 10.2.3  
Steady-State Partition Applied to Initial Distribution**

		Traditional										
		ND	FF	1	2	3	4	5	6	7	8	
Incoming		0.0813	0.1498	0.6719	0.0325	0.0176	0.0204	0.0098	0.0053	0.0033	0.0081	
X	D	0.1938	0.3222	0.2159	0.0967	0.0708	0.0522	0.0355	0.0297	0.0082	0.0063	
X	GR	0.0372	0.1077	0.1492	0.2709	0.3711	0.5148	0.5078	0.6220	0.7364	0.9667	
X	L	0.7690	0.5701	0.6349	0.6324	0.5580	0.4330	0.4567	0.3484	0.2554	0.0270	<b>TOTAL</b>
Of	D	0.0158	0.0483	0.1451	0.0031	0.0012	0.0011	0.0003	0.0002	0.0000	0.0001	0.2152
Total	GR	0.0030	0.0161	0.1002	0.0088	0.0065	0.0105	0.0050	0.0033	0.0024	0.0078	0.1637
	L	0.0625	0.0854	0.4266	0.0205	0.0098	0.0088	0.0045	0.0018	0.0008	0.0002	0.6211
												1.0000

		Non-Traditional										
		ND	FF	1	2	3	4	5	6	7	8	
Incoming		0.1302	0.0327	0.5191	0.0776	0.0495	0.0832	0.0194	0.0138	0.0097	0.0648	
X		0.1417	0.1890	0.1028	0.0354	0.0172	0.0133	0.0160	0.0130	0.0110	0.0076	
X		0.0289	0.0865	0.1315	0.2287	0.3222	0.4641	0.3322	0.4431	0.6272	0.8873	
X		0.8294	0.7244	0.7657	0.7359	0.6606	0.5226	0.6519	0.5439	0.3618	0.1051	<b>TOTAL</b>
Of	D	0.0252	0.0105	0.1121	0.0075	0.0035	0.0043	0.0007	0.0004	0.0001	0.0004	0.1648
Total	GR	0.0048	0.0035	0.0774	0.0210	0.0184	0.0428	0.0099	0.0086	0.0071	0.0627	0.2563
	L	0.1001	0.0186	0.3296	0.0491	0.0276	0.0360	0.0089	0.0048	0.0025	0.0017	0.5789
												1.0000

### 10.3 The Steady-State Model Applied to Degree Groupings

The transition matrix is reclassified by Degree Groupings alone. Initially, this transition matrix also included states by Classification Index. However, with 10 Classification Indexes, 7 Degree Groupings, and 3 reasons for becoming non-enrolled, this 73-by-73 matrix was too unwieldy, exceeding dimension limitations for some of Excel's matrix functions. After several alternative attempts to compress classification indexes into fewer states, it was determined that the transition matrix most representative of movement between data points would include only Degree Groups, and eliminate Classification Indexes entirely.

The partitioned steady state matrix, by age and Classification Index follows. The matrices required to develop the steady-state matrix for this grouping of data will also follow.



**Table 10.3.1**  
**Partitioned Transition Matrix by Degree Grouping**

		CURRENT									
		D	G	L	AS	BS	BR	HS	PRE	TR	UN
NEXT	D	1.0000			0.0065	0.0102	0.1470	0.0194	0.0720	0.0373	0.0847
	G		1.0000		0.1253	0.0679	-	-	0.0007	0.0022	0.0280
	L			1.0000	0.0782	0.1395	0.1994	0.3935	0.1839	0.2729	0.3459
	AS				0.4123	0.0831	0.0719	0.0290	0.1316	0.0778	0.0546
	BS				0.1919	0.5703	0.0678	0.0581	0.1318	0.1857	0.0805
	BR				0.0165	0.0127	0.3111		0.0335	0.0270	0.0091
	HS							0.4484			
	PRE				0.1247	0.0715	0.1608	0.0129	0.3985	0.1106	0.0833
	TR				0.0336	0.0383	0.0292	0.0258	0.0341	0.2761	0.0252
	UN				0.0110	0.0066	0.0126	0.0129	0.0139	0.0103	0.2885

**Table 10.3.2**  
**Steady-State Calculation by Degree Grouping**

**Identity Matrix**

1.0000						
	1.0000					
		1.0000				
			1.0000			
				1.0000		
					1.0000	
						1.0000

**Partition R of Transition Matrix**

0.4123	0.0831	0.0719	0.0290	0.1316	0.0778	0.0546
0.1919	0.5703	0.0678	0.0581	0.1318	0.1857	0.0805
0.0165	0.0127	0.3111		0.0335	0.0270	0.0091
			0.4484			
0.1247	0.0715	0.1608	0.0129	0.3985	0.1106	0.0833
0.0336	0.0383	0.0292	0.0258	0.0341	0.2761	0.0252
0.0110	0.0066	0.0126	0.0129	0.0139	0.0103	0.2885

**(I – R) Calculation**

0.5877	(0.0831)	(0.0719)	(0.0290)	(0.1316)	(0.0778)	(0.0546)
(0.1919)	0.4297	(0.0678)	(0.0581)	(0.1318)	(0.1857)	(0.0805)
(0.0165)	(0.0127)	0.6889	-	(0.0335)	(0.0270)	(0.0091)
-	-	-	0.5516	-	-	-
(0.1247)	(0.0715)	(0.1608)	(0.0129)	0.6015	(0.1106)	(0.0833)
(0.0336)	(0.0383)	(0.0292)	(0.0258)	(0.0341)	0.7239	(0.0252)
(0.0110)	(0.0066)	(0.0126)	(0.0129)	(0.0139)	(0.0103)	0.7115

**(I – R)<sup>-1</sup> Inverse of (I – R) Matrix**

2.0682	0.5673	0.4469	0.2137	0.6366	0.4863	0.3205
1.2386	2.8583	0.7019	0.4527	1.0098	1.0551	0.5833
0.1126	0.1008	1.5118	0.0266	0.1389	0.1164	0.0598
-	-	-	1.8129	-	-	-
0.6512	0.5296	0.6155	0.1706	1.9945	0.5388	0.3705
0.1988	0.2082	0.1498	0.1092	0.1847	1.4916	0.1152
0.0611	0.0504	0.0544	0.0458	0.0634	0.0516	1.4258

**Table 10.3.2 (continued)**  
**S Partition of Transition Matrix**

0.0065	0.0102	0.1470	0.0194	0.0720	0.0373	0.0847
0.1253	0.0679	-	-	0.0007	0.0022	0.0280
0.0782	0.1395	0.1994	0.3935	0.1839	0.2729	0.3459

**$S(I - R)^{-1}$  Calculation**

0.1020	0.0977	0.2868	0.0652	0.1906	0.1298	0.1686
0.3459	0.2674	0.1060	0.0592	0.1520	0.1378	0.1202
0.5521	0.6349	0.6072	0.8756	0.6574	0.7324	0.7112

**Final Steady-State Partition**

	AS	BS	BR	HS	PRE	TR	UN
D	0.1020	0.0977	0.2868	0.0652	0.1906	0.1298	0.1686
G	0.3459	0.2674	0.1060	0.0592	0.1520	0.1378	0.1202
L	0.5521	0.6349	0.6072	0.8756	0.6574	0.7324	0.7112
check	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The distribution of incoming students classified by Degree Groups is applied to the steady-state partition in order to obtain an overall graduation rate, over time.

**Table 10.3.3**  
**Steady-State Partition Applied to Initial Distribution**

		AS	BS	BR	HS	PRE	TR	UN	
Incoming		0.0732	0.1579	0.1542	0.0265	0.3839	0.0922	0.1120	
X	D	0.1020	0.0977	0.2868	0.0652	0.1906	0.1298	0.1686	
X	GR	0.3459	0.2674	0.1060	0.0592	0.1520	0.1378	0.1202	
X	L	0.5521	0.6349	0.6072	0.8756	0.6574	0.7324	0.7112	<b>TOTAL</b>
Of	D	0.0075	0.0154	0.0442	0.0017	0.0732	0.0120	0.0189	0.173
Total	GR	0.0253	0.0422	0.0163	0.0016	0.0583	0.0127	0.0135	0.170
	L	0.0404	0.1003	0.0936	0.0232	0.2524	0.0675	0.0796	0.657
		check							1.000

The steady-state partition applied to the initial distribution by degree group shows a long-term graduation rate of 17%, which is similar to overall graduation rates in previous versions of the model. By analyzing the individual groupings of the partition, it can be

shown that 34.59% of students seeking Associate degrees, and 26.74% of students seeking Bachelor degrees can be expected to graduate, given enough time. Less established degree groups such as Bridge and Undeclared have graduation rates of 10.60% and 12.02%. Of note is the Pre group with the largest percentage of incoming students, and a long-term graduation rate of 15.20%. Finally, of the students first admitted to this university as High School students, 5.92% can be expected to graduate from this same campus.

#### **10.4 Steady-State by Degree Group without Bridge Students**

One potential future impact on student demographics is the reduction or elimination of the Bridge program. Students who are found to have academic deficiencies will be required to participate in a community college program at a different university until those deficiencies are met.

Therefore, the last variation of the steady-state model will eliminate Bridge students from the incoming array that is applied to the Final Steady-State Partition. Elimination of incoming Bridge students will be the only change made to the model. The model does allow for students to move from any other degree group to the Bridge program, should this be necessary. This movement will still be allowed in the model. However, the new model will not allow students who are initially classified as Bridge students to be admitted.

**Table 10.4.1**  
**Steady-State Partition Applied to Initial Distribution without Bridge Students**

		AS	BS	BR	HS	PRE	TR	UN	
Incoming		0.0866	0.1867	-	0.0313	0.4539	0.1090	0.1324	
X	D	0.1020	0.0977	0.2868	0.0652	0.1906	0.1298	0.1686	
X	GR	0.3459	0.2674	0.1060	0.0592	0.1520	0.1378	0.1202	
X	L	0.5521	0.6349	0.6072	0.8756	0.6574	0.7324	0.7112	<b>TOTAL</b>
Of	D	0.0088	0.0182	-	0.0020	0.0865	0.0142	0.0223	0.152
Total	GR	0.0300	0.0499	-	0.0019	0.0690	0.0150	0.0159	0.182
	L	0.0478	0.1185	-	0.0274	0.2984	0.0799	0.0942	0.666

check 1.000

In the previous model, the Bridge students comprised approximately 15% of incoming students. Bridge students are also shown to have an expected graduation rate of 11%. However, the elimination of these less established students still only causes the overall graduation rate to rise to 18.2% from 17%.

## 11. Summary and Analysis of Results

In this project, retention and graduation rates for several under analyzed student classifications have been developed for the purpose aiding university retention committees in developing guidelines for increasing retention.

### 11.1 First-Year Retention

It has been shown that incoming freshmen, even those with previous college experience, are indeed more at risk for dropping out before the next academic year than higher classification levels of students. It has also been shown that even newly enrolled students with higher classification indexes have a significantly higher first-year retention rate

than newly enrolled freshmen. This suggests that higher retention rates are associated with student capability and experience rather than continuity at a single campus.

Further, first-year retention rates also indicate that while students in good academic standing are retained at a significantly higher rate than students on academic probation, students in good standing are still retained at a rate of only 46%. This suggests reasons beyond academic performance for dropping out.

Traditionally aged students have a lower first-year retention rate of 41% compared to non-traditionally aged students at 49%. This lends some credence to the often heard notion that more mature students perform better academically.

Full-time students are retained at a rate of 49% compared to 39% for part-time students. This supports the statements in the literature, although it does not support the objective of attracting more part-time students to become full-time students.

Finally, students who are admitted directly into their degree programs have much higher retention rates at 72% for Associate degree students, and 61% for Bachelor degree students, than students admitted into Pre or Bridge programs at 38% and 13%. Certainly students admitted directly into their majors are more capable students than Pre or Bridge students, but discouragement is also a factor for students who have semesters of deficiencies to remedy before they can begin their two- or four- year programs.

## **11.2 Graduation Rates**

Both six-year and long-term graduation rates for all students range from 17% to 25%. Patterns in graduation rates for various student classifications are unsurprisingly similar to first-year retention rates. Juniors and Seniors are more likely to progress to graduation than

Freshmen, even when the Juniors and Seniors are newly enrolled students. Non-traditionally aged students graduate at a somewhat higher rate within the range than traditionally aged students, but surprisingly, a higher percentage of part-time students graduate than full-time.

Finally, the highest graduation rates of 35% and 27% are seen for students in Associate and Bachelor degree programs rather than less established programs.

## **12. For Further Consideration**

Retention and graduation rates for specific student groups show who is being retained and graduated, but not why this is occurring. Further research into the issue of retention will need to take a more qualitative approach.

“Calling programs,” in which representatives of the university contact students for exit interviews upon graduation, and to encourage enrollment by students at risk of dropping out are commonly used at this campus. A suggestion for further study would be to institute a randomized calling program to survey a sample of all students in order to obtain information about needs in the areas of motivation and support.

Finally, retention rates for students using existing campus student support programs can be developed using the methods described in this project.

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